The Texas Education Agency's

Lighthouse Initiative for Mathematics Classrooms

Texas Essential Knowledge and Skills (TEKS)

and the

*Advanced Placement Program**

Why a lighthouse? In the summer of 2001, a document was produced to help teachers in grades 7–12 understand how the Texas Essential Knowledge and Skills (TEKS) for English/language arts and reading aligned with Advanced Placement Program* English objectives. Being the creative people that they are, the English teachers chose a lighthouse as a metaphor for their project.

The great beacon at the top of the lighthouse should make safe the navigation of ships to their appointed harbors. –Unknown

Just as a lighthouse must sit on bedrock, the TEKS for mathematics form the foundation for curriculum, instruction, and assessment in Texas mathematics classrooms. The classroom curriculum should provide students with a challenging framework of lessons designed to help them "navigate" important, often difficult, material.

The Lighthouse Initiative for Mathematics Classrooms was created by teachers for teachers to help plan and coordinate mathematics Pre-AP* programs. The goal of the Lighthouse Initiative is to enable students from all walks of life to participate in AP* classes, thus lighting the way for future academic success.

Please note: This Web site was updated in fall 2007 to reflect the revisions to the mathematics TEKS.

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Background

Please follow these links to learn a little more about how the Lighthouse program got started in the realm of mathematics.

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- The Goals of the Pre-AP* Mathematics Program
- The Role of Middle School in the AP* Program
- The Growth of the AP* Program in Mathematics in Texas
- The IB Programme and the TEKS
- AP* Statistics: Providing Students Another Opportunity for Excellence

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Introduction to the Lighthouse Initiative for Mathematics Classrooms

One day in the summer of 2001, I was sitting at my computer when I got a phone call from someone at the Texas Education Agency. She told me about a document that had been produced to show how the English/language arts and reading TEKS for grades 7–12 were aligned to Pre-AP* objectives and asked if I was interested in working on a similar project for the mathematics TEKS. I was impressed with the document that the English folks had produced and with teachers' response to it. As an AP* teacher for over a decade, I have a strong belief in the power of the AP program to transform students' lives and open up educational opportunities for them. I am also a big believer in the necessity of a well-planned and coordinated Pre-AP program to enable students from all walks of life to participate in AP classes, if they choose to do so. I knew that producing this document would be a monumental responsibility, but for some reason, agreed to the task.

The first and smartest thing that I have ever done was in convincing/cajoling the people whose names you see in the contacts section to agree to work on it with me. They come from districts all over Texas, teach students from a wide variety of backgrounds and from various grade levels, and have different experiences with the AP program. What they have in common are a strong content background in mathematics and a commitment to opening up the pipeline to advanced mathematics for all Texas students. At our first meeting, we agreed that there were as many ways to build an effective Pre-AP mathematics program as there are districts in Texas. This document does not attempt to prescribe a Pre-AP course description or to dictate to teachers how they should teach their classes. It is intended to provide teachers with some guidelines and ideas about how they can address the TEKS while simultaneously introducing the skills, concepts, and work habits necessary for success in AP mathematics classes. On this site, you will find a set of goals that the Mathematics Lighthouse committee agreed all effective Pre-AP mathematics programs would have in common. We also generated a list of things that ARE NOT indicative of a good Pre-AP program:

- Pre-AP mathematics classes are not merely re-named honors classes. Much of what you see on this website can be used with students in a regular program. Our purpose is not to "honor" anyone but to open up opportunities for as many students as possible.
- Pre-AP mathematics classes do not just teach the same old things one year earlier. Pre-AP is not just about accelerating students or topics. It is about teaching with a particular goal in mind and making instructional decisions that will support that goal.

We hope that you will find this Web site useful. The process of producing it certainly had a profound effect on all of those who served on the committee. We understand how important the Pre-AP program is and how daunting it is to get started on something like this. At the same time, we are excited by what we have learned. Since there are so many places where Pre-AP is achieving its goal of providing access to advanced mathematics for more students, we are confident that it can be done in classrooms throughout Texas, that it can be done by us, and that it can be done by **you**.

Dixie Ross

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The Goals of the Pre-AP* Mathematics Program

Pre-AP* mathematics classes should do the following:

- Prepare growing numbers of students, especially those traditionally underrepresented in AP* courses, for the challenges offered by the Advanced Placement Program*
- Introduce skills, concepts and assessment methods to prepare students for success in AP and other challenging courses
- Require students to work with functions represented in a variety of ways (graphical, numerical, analytical, or verbal) and understand the connections among these representations
- Encourage students to develop their communication skills in mathematics to be able to read and interpret problem situations and explain solutions to problems both orally and in well-written sentences
- Provide students with multiple opportunities to model a written description of a physical situation with a function or to determine appropriate functions to match data sets that they are given or that they develop through experimentation or research
- Make regular use of technology to help solve problems, experiment, interpret results, and verify conclusions
- Require students to determine the reasonableness of their solutions, including sign, size, relative accuracy, and units of measurement
- Help students to develop an appreciation of mathematics as a coherent body of knowledge and as a human accomplishment
- Be a part of a well-planned and coherent curriculum so that teachers can build upon knowledge and skills that students have acquired in previous courses and can prepare students for subsequent courses
- Allow students to develop the work ethic and habits of mind that are necessary for success in the Advanced Placement Program and in other challenging mathematics programs

Many of these goals have been adapted from The Teacher's Guide to AP Calculus *and the Pre-AP brochure from the College Board.*

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The Role of Middle School in the Advanced Placement Program*

Sometimes I wonder who is more challenged in the Pre-AP* mathematics program—the teacher or the students. The Pre-AP mathematics program begins in middle school and targets students who are ready for a more rigorous curriculum than is normally taught. The Pre-AP courses these students take should prepare them for the subsequent high school courses that lead to AP* Calculus or AP Statistics.

I began teaching 6th grade Pre-AP mathematics in the fall of 1995. Initially, I believed instruction merely involved moving through the prescribed curriculum at an accelerated pace. The Pre-AP curriculum at the middle school level needs to cover the TEKS for grades 6-8, the standards for Pre-Algebra and the Algebra I TEKS. I soon realized that racing through the curriculum with no additional depth leads to frustration as well as an absence of meaningful learning. It has been a challenge for me to re-evaluate how I teach mathematics. I must present problems in ways that inspire students, cover several TEKS or topics in one lesson, and broaden learning beyond the grade 6 accelerated curriculum.

Rigorous and high standards upheld in a supportive environment are the hallmarks of a good Pre-AP program. Building a strong foundation in the early Pre-AP classes leads to success in AP classes. My connection with a vertical team in my school district, attending Pre-AP conferences, and my work with the Lighthouse Initiative for Mathematics Classrooms has been instrumental in helping me understand how best to challenge my students. I have used several modified calculus and statistics problems in my classes with much success. My students' excitement about being able to do such work is inspiring. I frankly have been amazed at how well they do and how much learning goes on when class activities apply to their lives. I have also found that not just Pre-AP classes, but all classes, have had some of the same successes with the activities and enjoyed the challenge of exploring these topics. I believe teaching Pre-AP mathematics has made me a better teacher in all of my classes.

The Pre-AP experience often presents a struggle that bewilders parents and students. In the NAASP Bulletin (February 1997), Suzanne Sutton commented that

> "Struggling in mathematics is not the enemy, any more than sweating is the enemy in basketball; it is part of the process, and a clear sign of being in the game. Mathematics asks our students to think in ways they are not used to . . . A rigor of thinking and clarity of expression is demanded that will stretch them beyond familiar styles."

We, as educators, play a significant role in communicating with students, parents, and the community about the value of the struggle. The "no effort A" is not enough. It is the struggle to go that extra step that teaches the discipline of perseverance—a discipline so needed by our students in schoolwork and in life activities. The maturity level of some students often prevents them from entering the Pre-AP program in the 6th grade, but a strong regular education curriculum will allow movement of students to Pre-AP mathematics classes in a later grade. Students, parents, and teachers at all grade levels should be fully aware of the multiple avenues of access to the AP program.

I believe a challenging Pre-AP program sets goals for students, something missing in many of their lives. It is fulfilling for me as a teacher to see students walk into class, get excited about mathematics, and connect with the learning process. All mathematics teachers at the middle school level have a challenge before them. They must prepare students for higher education and in such a way that it will also prepare them to become lifelong learners and contributors to society. The Pre-AP program provides tools that teachers can use in meeting this challenge and a framework that will help them to improve their instruction for all students.

Donna Enochs

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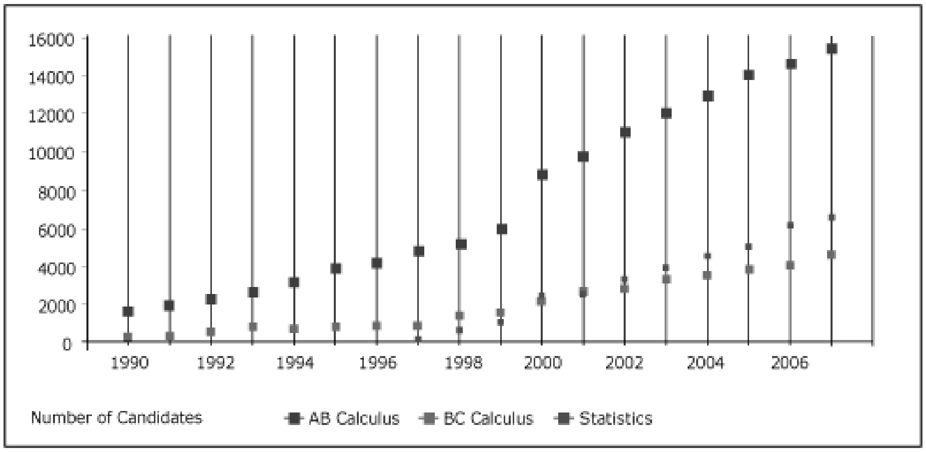
The Growth of the Advanced Placement Program* in Mathematics in Texas

The Advanced Placement Program* in mathematics has been in place since 1955. In its initial years, many of those who took advantage of the program were from exclusive, expensive private schools or from larger suburban public high schools serving upper middle class communities. More recently, the AP* program has become a tool to promote equitable access to excellence in academic achievement for all students. In 1993, the Texas legislature passed the Advanced Placement Incentive Program to provide funding to offset examination fees for low-income students and to provide training through summer institutes for AP teachers. In 1997, AP participation rates and success rates for Texas high schools were added to the Academic Excellence Indicator System, most well known for ranking Texas high schools based on Texas Assessment of Knowledge and Skills (TAKS) performance. In 1998, the Texas Education Agency adopted the AP course descriptions from the College Board as part of the new Texas Essential Knowledge and Skills (TEKS). That same year, the Report of the Governor's Texas Science and Technology Council, under then Governor George Bush, set a goal to triple the number of AP exams with passing scores (in all subject areas) to 100,000 by the year 2002 and to increase the number of school districts in Texas with AP programs to 100 percent. In 1999, the Texas legislature increased funding for the AP Incentive Program to offset AP exam costs for all Texas public school students and to provide schools with successful exam performance with funding to be used at their discretion to further improve their AP programs. In the summer of 2002, the Texas Education Agency began to reimburse school districts for the cost of College Board-endorsed summer institutes for Pre-AP* high school teachers. Four years later, in 2006, the cost for middle school teachers to attend summer institutes was also covered.

Due to these initiatives, and the incredible efforts of dedicated Texas mathematics teachers, participation in AP mathematics programs by Texas public school students has increased by over 1005% since 1990. We now have more students earning the highest possible score of 5 on AP mathematics exams than were even taking the exams in 1990. Despite this enormous progress, we still have a long way to go, particularly in assuring that rural, low-income, and minority students in Texas have the same access to outstanding AP mathematics programs as do their counterparts in more affluent communities. The Pre-AP program is an important component in providing that access. Students who have participated in a coherent and articulated mathematics curriculum specifically designed to emphasize the skills, concepts, and work habits necessary for success in calculus and statistics will be best prepared for the AP program in mathematics and for other post-secondary educational opportunities.

The main purpose of the Lighthouse Initiative for Mathematics Classrooms is to provide Texas teachers with guidance in how they can provide access for all students to Pre-AP mathematics objectives while simultaneously addressing the TEKS. If this purpose is achieved, we will continue to see growth in the numbers of students participating in the AP mathematics program and in their success as measured by AP exam performance. The AP program will continue to maintain its reputation for promoting academic achievement while also serving as a tool to promote equity in educational opportunity for all Texas students.

GROWTH OF AP
MATHEMATICS IN TEXAS, 1990-2007



Year	AB Calculus	BC Calculus	Statistics
1990	1,808	482	
1991	2,094	553	
1992	2,334	704	
1993	2,806	911	
1994	3,330	840	
1995	4,092	1,004	
1996	4,296	1,036	
1997	4,953	1,075	293
1998	5,282	1,515	786
1999	6,160	1,701	1,214
2000	8,447	2,300	2,164
2001	9,977	2,815	2,720
2002	11,131	3,020	3,417
2003	12,154	3,401	4,043
2004	13,167	3,743	4,658
2005	14,142	3,964	5,167
2006	14,773	4,208	6,320
2007	15,932	4,817	6,667

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The International Baccalaureate Programme and the TEKS

The Lighthouse Initiative for Mathematics Classrooms focuses on the Pre-AP* program and the mathematics TEKS for the state of Texas. One of the goals of the Pre-AP program is for students to develop the skills and learning techniques necessary for success in college. Another program, the International Baccalaureate (IB) Programme, has similar goals and has been adopted by some Texas schools either in addition to or as an alternative to the Advanced Placement Program.*

These Lighthouse resources can be used for the IB Middle Years Programme. The ideas and suggestions put forth on this site are consistent with the goals and mission statements of both AP* and IB.

For more information please visit the IB Web site at www.ibo.org.

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AP* Statistics: Providing Students Another Opportunity for Excellence

Since its inception in 1997, the Advanced Placement Program (AP)* in statistics has set records for growth in the number of exams taken in a new AP subject. From elementary school through middle school, the mathematics curriculum framework in the state of Texas, the Texas Essential Knowledge and Skills or TEKS, emphasizes concepts about chance, data collection, and data exploration that are critical for developing the prerequisite skills for AP Statistics. Traditionally, the secondary school mathematics curriculum has been designed to prepare students for success in calculus. Now, the mathematics curriculum must prepare students for success in both calculus and statistics. The new state requirement of four years of high school mathematics offers an opportunity for AP Statistics to grow even more. Students may use AP Statistics to satisfy their fourth year of mathematics requirements. While some students may choose to take either AP Calculus or AP Statistics, ideally, students will take both, creating graduates with an extensive and integrated understanding of mathematics.

The explosion in the use of statistics in the last 50 years must strike anyone who reads the newspaper, listens to the radio, watches television, or checks out the World Wide Web. For some time, knowledge of statistics has been important for decision-makers and scientists. Surveys of public opinion (such as the Gallup political polls), of behavior (such as the Nielsen ratings of television viewing), of labor force characteristics (such as the Current Population Survey) increasingly help determine government policy and the choices available to consumers. Further, much recent progress in fields such as medicine, engineering, the life and physical sciences, and the social sciences can be attributed to the use of statistics.

Statistics, the science of data, is integral to the methodology of a wide variety of disciplines, many of which extend far beyond the "calculus-based" sciences. For example, it is an essential tool in all of the behavioral, biological, and social sciences. Students of biology, business, engineering, psychology, and many other disciplines will inevitably encounter statistics in their college studies. Introductory courses in statistics are taught as part of the undergraduate curriculum in many of these fields, as well as in mathematics. In fact, some postsecondary institutions see statistics as so essential to the quantitative reasoning skills of all graduates that statistics courses have become part of the required general education program. A recent survey indicated that introductory statistics enrollments in statistics or mathematics departments numbered about 240,000 in fall 2000, up about 16 percent from fall 1995. Some speculate that this number would double if statistics enrollments in all departments were counted.

Throughout this site, you will find examples showing how one can take the ideas and concepts of the AP Statistics program and integrate them into the TEKS for mathematics. By doing so, schools will be able to provide their students with another opportunity to access higher level mathematics and thus be better prepared for college and/or life experiences.

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Content

One of the very best ways for Pre-AP* mathematics teachers to understand the goals, skills, and work habits necessary for success in AP* Calculus and AP Statistics is to examine released AP problems and talk with AP mathematics teachers in their district about where their students encountered difficulty. In this section of the Lighthouse website, we have chosen two AP free-response problems (the ones that require students to show their work and that are graded by actual humans)—one from calculus and one from statistics. Following each problem are several variations written by Pre-AP teachers to address some common areas of difficulty. You will see the actual worksheets that teachers made, so there will be different styles and formats. Figures are often hand drawn so they might be rough and not drawn to scale. Each variation includes a teacher version with commentary and answers followed by a clean copy that could be used with students. Examining actual AP problems and the difficulties that students had with them should be a regular and ongoing part of a mathematics vertical team's discussions.

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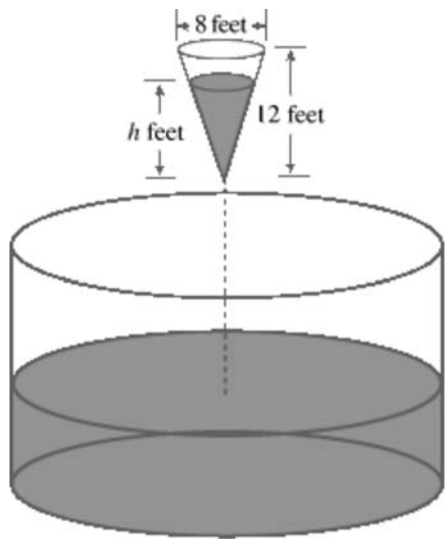
AP* Calculus AB Problem 5, 1995

This problem was one of the six free-response questions on the 1995 AP* Calculus AB exam. Students had a great deal of difficulty in recognizing the similarity of the water in the cone to the cone figure itself. In Part A, they seemed to have difficulty with the wording of the problem. Had it said, "Solve for V in terms of h," a greater number of students probably would have solved the equation correctly. Students also had difficulty in working with the volume formulas and with the simple symbolic manipulation that was required. All of these difficulties occurred well before the students had a chance to reach the calculus stage of the problem. The concepts of proportionality, similarity, volume, and symbolic manipulation are introduced as early as middle school, but perhaps could be approached a little differently in a Pre-AP* situation. In addition to examining the similarity of plane figures, for example, Pre-AP students could also examine the similarity of three-dimensional figures and embedded figures. Instead of just using formulas to calculate area or volume, there could be more emphasis on manipulating the formulas and expressing them in different forms. The same TEKS would be addressed but in a way that will better prepare students for the AP experience.



5. As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of (h-12) feet per minute. (The volume V of a cone with radius r and height h is $V = 1/3\pi r^2 h$.)

(a) Write an expression for the volume of water in the conical tank as a function of h.

(b) At what rate is the volume of water in the conical tank changing when h = 3? Indicate units of measure.

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when h = 3? Indicate units of measure.

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Three Middle School Activities Based on Problem 5

Based on AP* Calculus AB Problem 5, 1995

Middle School Activity 1, Based on Problem #5

TEKS addressed: (6.8)(A); (6.10)(D); (6.12)(A)

This Pre-AP* activity introduces the calculus topic rate of change. It was conducted in a Pre-AP grade 6 classroom. Students began drawing graphs from data collected by pouring rice into containers and then moved to a more abstract level by drawing graphs from pictures of containers. The purpose of this activity was to have students think about the rate at which the height of rice in various containers changed depending upon the shape of the containers.

Teacher Guide

Sample A:

- Student Sample A, Graph
- Student Sample A, Graph Summary
- Student Sample A, Worksheet Part 1
- Student Sample A, Worksheet Part 2

Sample B:

- Student Sample B, Graph
- Student Sample B, Volume Graph
- Student Sample B, Written Summary

Middle School Activity 2, Based on Problem #5

TEKS addressed: (6.2)(C); (6.3)(C); (6.8)(B); (7.3)(B); (8.8)(A); (8.9)(B); (8.10)(A); (8.10)(B)

This is a good Pre-AP worksheet because it addresses the difficulty that students have with the similarity of three-dimensional figures. It addresses several TEKS simultaneously and goes well beyond what is covered in most textbooks. Notice that in problems 1-3, surface area and volume for each figure can be calculated directly. In problem 5, you can easily find surface area and volume of the first figure and then set up proportions to find values for the second figure.

Student Worksheet
Teacher Guide and Answer Key

Middle School Activity 3, Based on Problem #5

TEKS addressed: (6.1)(B); (7.2)(F); (7.3)(B); (8.7)(A); (8.8)(C); (8.9)(B); (8.10)(B)

This is a good Pre-AP worksheet because it builds upon and extends skills that students have developed in middle school and Algebra I (using geometric formulas, solving literal equations, symbolic manipulation, function notation) while simultaneously addressing some TEKS for high school geometry. It does so in a way very different from that seen in most textbooks. Most importantly, the context and wording is based upon an actual AP problem with which the students had difficulty.

Student Worksheet
Teacher Guide and Answer Key

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Teacher Guide, Middle School Activity 1

TEKS covered by this activity: 6.8A, 6.10D, and 6.12A

This Pre-AP* activity introduces the calculus topic rate of change. It was conducted in a Pre-AP grade 6 classroom. Students began drawing graphs from data collected by pouring rice into containers and then moved to a more abstract level by drawing graphs from pictures of containers. The purpose of this activity was to have students think about the rate at which the height of rice in various containers changed depending upon the shape of the containers. Students were divided into groups of three to four people. Each group used three different shaped containers and took measurements of the heights of rice in their containers.

The students in each group accomplished the following activities:

1. Measured fixed quantities of rice using a coffee or laundry detergent scoop. (The students poured the rice into the container and then measured, recorded, and graphed the resulting height of the rice after each scoop. This process was repeated for two other shaped containers.)
2. Displayed their three graphs on the wall for student discussion
3. Discussed the different slopes in each of their graphs and how the slopes relate to the rate of change of the height
4. Selected three different containers and drew individual graphs* based on the data displayed on the wall
5. Used worksheets* to continue their study of graphing and rates of change
6. Summarized their work* by answering the following questions:
    a. Which of the containers has the slowest rate of change of rice height with respect to volume?
    b. What general statement can you make about the relationship between the shape of the container and the rate at which the rice changes with respect to volume?

Reflections on the activity: In thinking back over the activity, the instructions should stress that the students explain, in complete sentences, the reasoning behind their graphs or why they matched a graph to a particular container. All axes should have been labeled and have appropriate scales.

*See student samples of work associated with this activity.

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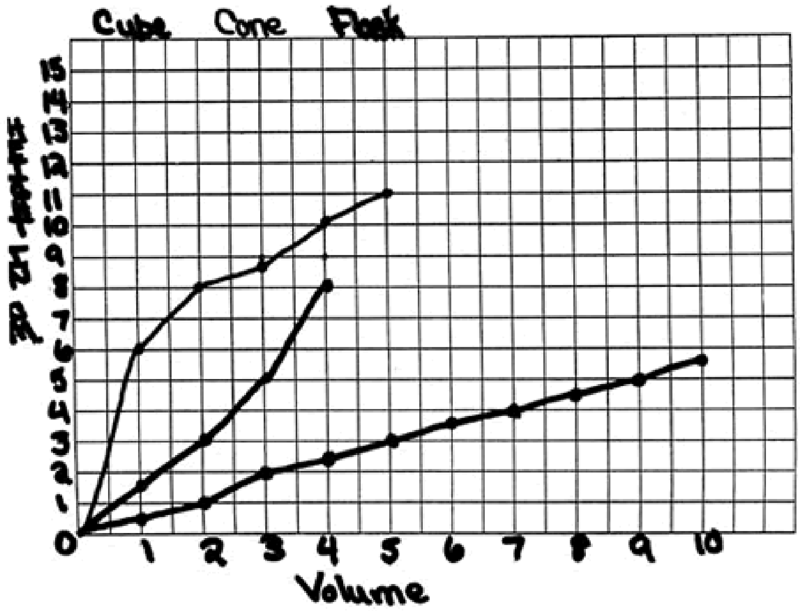
Student Sample A, Graph

Based on AP* Calculus AB Problem 5, 1995

STUDENT SAMPLE A



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Student Sample A, Graph Summary

Based on AP* Calculus AB Problem 5, 1995

STUDENT SAMPLE A

Graph Summary

① Which of the bottles has the slowest rate of change of rice heigth with respect to volume? cube

② What statement can you make about relationship between the shape of bottle and rate of rice heigth changes with respect to volume. I found out that the wider the circumference the slower rate of rice change. The same thing relates with the cube, the wider the perimeter the slower rate of change. But in the narrower parts of the bottles the rice heigth rises.

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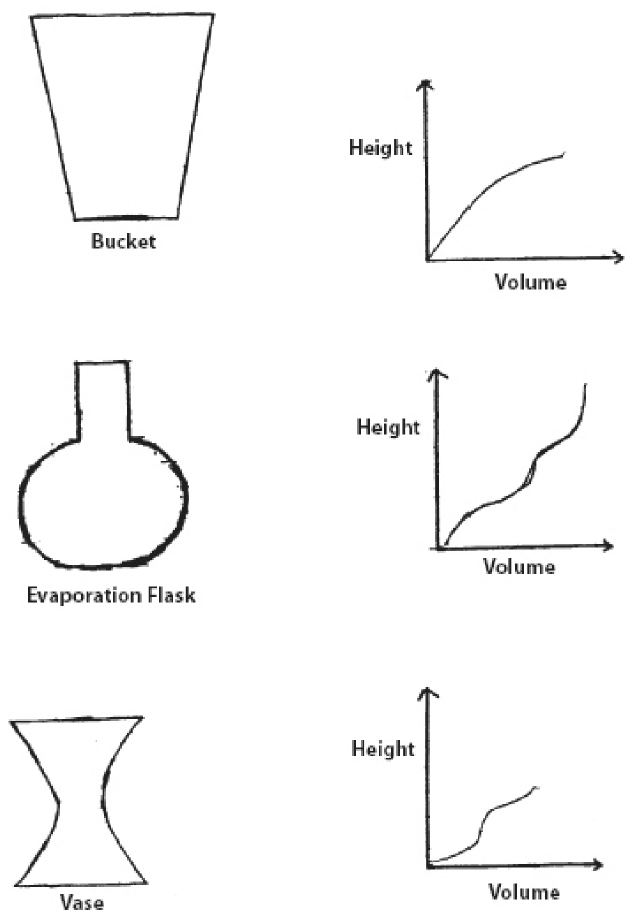
Student Sample A, Worksheet Part 1

Based on AP* Calculus AB Problem 5, 1995

STUDENT SAMPLE A

Sketch your own graph for each container that shows the height of the rice versus the volume of the rice that has been poured into the bottle.



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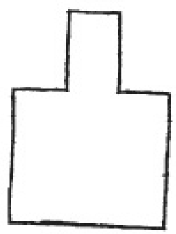
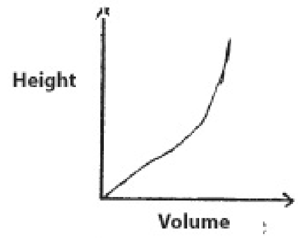
Student Sample A, Worksheet Part 2

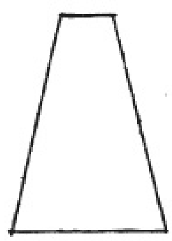
Based on AP* Calculus AB Problem 5, 1995

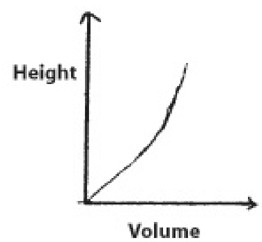
STUDENT SAMPLE A

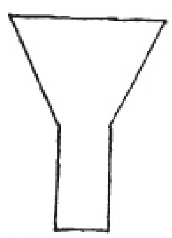
Sketch your own graph for each container that shows the height of the rice versus the volume of the rice that has been poured into the bottle.

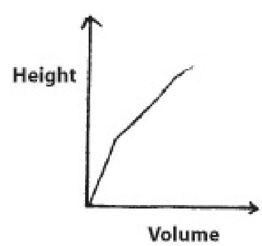

Ink Bottle




Conical Flask




Plugged tunnel



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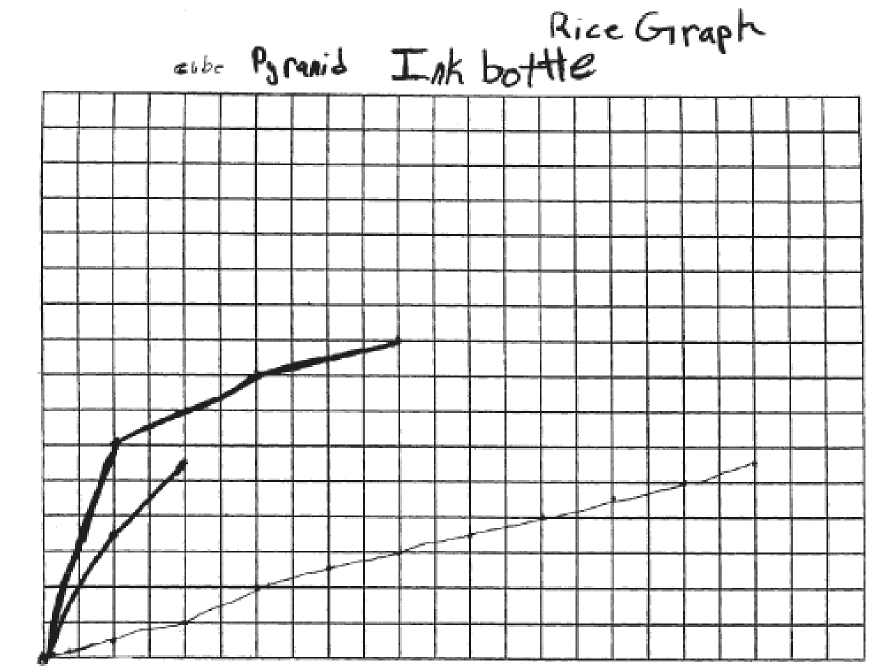
Student Sample B, Graph

Based on AP* Calculus AB Problem 5, 1995

STUDENT SAMPLE B



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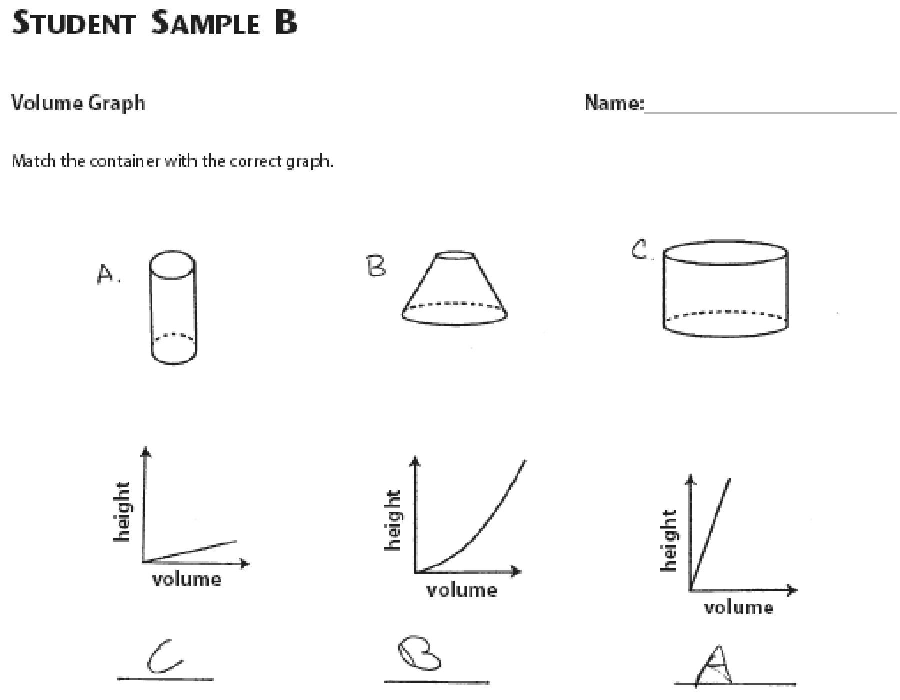
Student Sample B, Volume Graph

Based on AP* Calculus AB Problem 5, 1995



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Student Sample B, Written Summary

Based on AP* Calculus AB Problem 5, 1995

STUDENT SAMPLE B

1) Which of these bottles has the slowest rate of change of rice hight with respect to volume?
a> The cube, because it has an equal perimeter all the way up which makes a gradual increase.

2) What general statements can you make about the relationship between the shape of the bottle and the rate at which the rice hight changes with respect to volume?
What I learned was that the more equal perimeter or circumference the more gradual increase you would have. Also the slsinnier the bottle the higher and quicker the increase and the wider the bottle the longer the increase.

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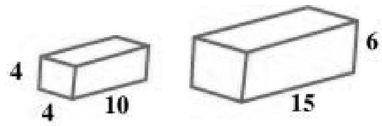
Student Worksheet, Middle School Activity 2

In each of the problems below, there are pairs of similar figures. Determine the surface area and volume of each figure. Then determine the ratios (in lowest terms) of the sides, the surface areas, and the volumes for each pair of figures.

1.

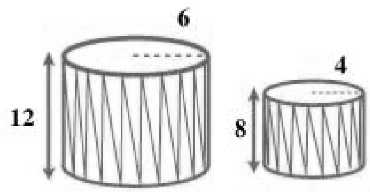


ratio of sides ____

S.A. ____ **S.A.** ____ **ratio of S.A.** ____

Volume ____ **Volume** ____ **ratio of V** ____

2.

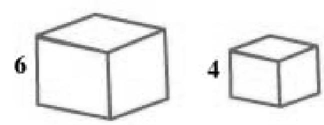


ratio of sides ____

S.A. ____ **S.A.** ____ **ratio of S.A.** ____

Volume ____ **Volume** ____ **ratio of V** ____

3.

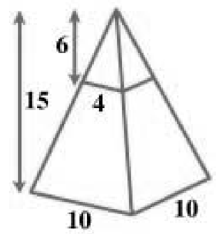


ratio of sides ____

S.A. ____ **S.A.** ____ **ratio of S.A.** ____

Volume ____ **Volume** ____ **ratio of V** ____

4.

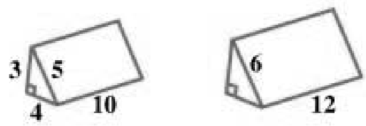


ratio of sides ____

S.A. ____ S.A. ____ ratio of S.A. ____

Volume ____ Volume ____ ratio of V ____

5.

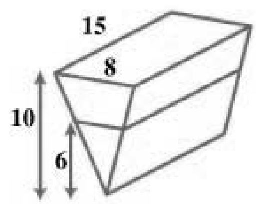


ratio of sides ____

S.A. ____ S.A. ____ ratio of S.A. ____

Volume ____ Volume ____ ratio of V ____

6.



ratio of sides ____

S.A. ____ S.A. ____ ratio of S.A. ____

Volume ____ Volume ____ ratio of V ____

7. What is the relationship between the ratio of the surface areas and the ratio of the sides? Why is this a reasonable result?

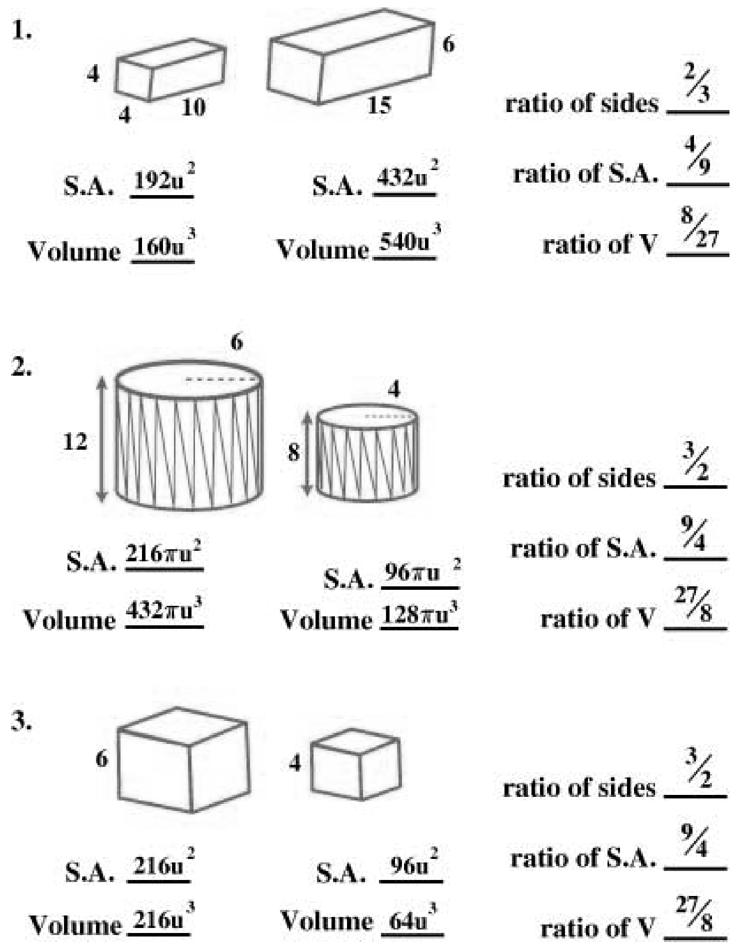
8. What is the relationship between the ratio of the volumes and the ratio of the sides? Why is this a reasonable result?

[printer-friendly]

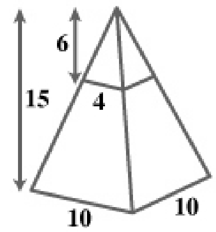
Teacher Guide and Answer Key, Middle School Activity 2

Based on AP* Calculus AB Problem 5, 1995

This is a good Pre-AP* worksheet because it addresses the difficulty that students have with the similarity of three-dimensional figures. It addresses several TEKS simultaneously and goes well beyond what is covered in most textbooks. Notice that in problems 1-3, surface area and volume for each figure can be calculated directly. In problem 5, you can easily find surface area and volume of the first figure and then set up proportions to find values for the second figure. In each of the problems below, there are pairs of similar figures. Determine the surface area and volume of each figure. Then determine the ratios (in lowest terms) of the sides, the surface areas, and the volumes for each pair of figures.



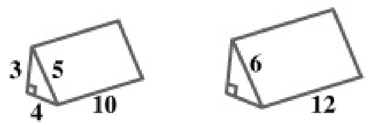
4.



ratio of sides $\frac{2}{5}$

S.A. ___	S.A. ___	ratio of S.A. $\frac{4}{25}$
Volume $32u^3$	Volume $500u^3$	ratio of V $\frac{8}{125}$

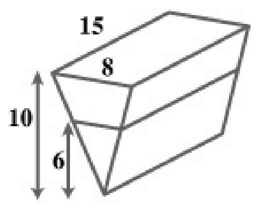
5.



ratio of sides $\frac{5}{6}$

S.A. 132	S.A. 190.08	ratio of S.A. $\frac{25}{36}$
Volume 60	Volume 103.68	ratio of V $\frac{125}{216}$

6.



ratio of sides $\frac{5}{3}$

S.A. ___	S.A. ___	ratio of S.A. $\frac{25}{9}$
Volume $600u^3$	Volume $129.6u^3$	ratio of V $\frac{125}{27}$

7. What is the relationship between the ratio of the surface areas and the ratio of the sides?

The ratio of the surface area is the square of the ratio of the sides.

Why is this a reasonable result?

Because area is measured in square units.

8. What is the relationship between the ratio of the volumes and the ratio of the sides?

The ratio of the volumes is the cube of the ratio of the sides.

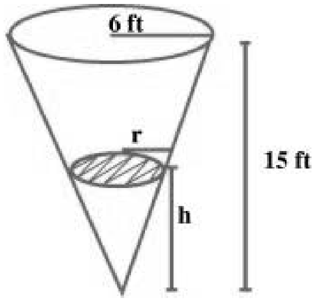
Why is this a reasonable result?

Because volume is measured in cubic units.

[printer-friendly]

Student Worksheet, Middle School Activity 3

Based on AP* Calculus AB Problem 5, 1995



Oil is being stored in a conical storage tank having a radius of 6 feet and a height of 15 feet.

1. What is the capacity of the storage tank?

2. What is the ratio of the tank radius to its depth?

3. If the height of the oil in the tank is 10 ft., what is the radius of the oil in the tank?

4. What is the volume of the oil in the tank?

5. What is the ratio of the volume of the oil to the capacity of the tank?

6. What is the ratio of the radius of the oil to the radius of the tank?

7. What is the relationship between your answers to #5 and #6?

8. What is the area of the surface of the oil in the tank?

9. If the oil currently in the tank is pumped out at a rate of 2 cubic feet/minute, how long will it take to empty the tank?

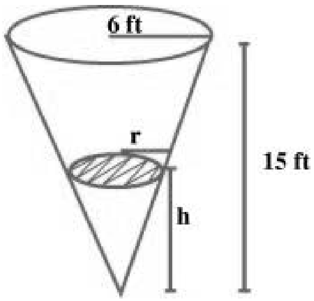
10. If the oil currently in the tank is pumped into another tank that is a right cylinder with a radius of 8 feet and a depth of 12 feet, what will be the depth of the oil in this new tank?

[printer-friendly]

Teacher Guide and Answer Key, Middle School Activity 3

Based on AP* Calculus AB Problem 5, 1995

This is a good Pre-AP* worksheet because it expects students to combine many different concepts and skills responding to a single scenario, much like an AP* test question does. Students have to be able to interpret the questions and select appropriate strategies. There are opportunities to discuss things pertaining to units and the different forms that the answers may take.



Oil is being stored in a conical storage tank having a radius of 6 feet and a height of 15 feet.

1. What is the capacity of the storage tank?

$180\pi \text{ ft}^3$

2. What is the ratio of the tank radius to its depth?

$\frac{r}{h} = \frac{2}{5}$

3. If the height of the oil in the tank is 10 ft, what is the radius of the oil in the tank?

$r = 4$ ft

4. What is the volume of oil in the tank?

$\frac{160\pi}{3} \text{ ft}^3$

5. What is the ratio of the volume of the oil to the capacity of the tank?

$\frac{8}{27}$

6. What is the ratio of the radius of the oil to the radius of the tank?

$\frac{2}{3}$

7. What is the relationship between your answers to #5 and #6?

#5 is the cube of #6

8. What is the area of the surface of the oil in the tank?

$16\pi \text{ ft}^2$

9. If the oil currently in the tank is pumped out at a rate of 2 cubic feet/minute, how long will it take to empty the tank?

$\frac{80\pi}{3}$ minutes or 83.776 minutes or 1 hour 23 minutes and 47 seconds

10. If the oil currently in the tank is pumped into another tank that is a right cylinder with a radius of 8 feet and a depth of 12 feet, what will be the depth of the oil in this new tank?

$\frac{5}{6}$ **ft or .833 ft or 10 inches**

[printer-friendly]

For more information, please send an e-mail to Curriculum@tea.state.tx.us

Two Geometry Activities Based on Problem 5

Based on AP* Calculus AB Problem 5, 1995

Geometry Activity 1 Based on AP* Calculus AB, Problem 5, 1995

TEKS addressed: (G.7)(A); (G.8)(A); (G.8)(D); (G.11)(B); (G.11)(C)

This is a good Pre-AP* worksheet because it builds upon and extends skills that students have developed in middle school and Algebra I (using geometric formulas, solving literal equations, symbolic manipulation, function notation) while simultaneously addressing the TEKS listed above for high school geometry. It does so in a way very different from that seen in most textbooks. Most importantly, the context and wording is based upon an actual AP* problem with which the students had difficulty.

Teacher Guide and Answer Key
Student Worksheet

Geometry Activity 2 Based on AP* Calculus AB, Problem 5, 1995

TEKS addressed: (a)(2); (a)(4); (a)(6); (G.1)(B); (G.5)(B)
Vocabulary includes literal equations and delta form of a slope equation.

This makes a good Pre-AP geometry problem because students must apply many concepts in one problem. Teachers may choose to teach parts of this problem at different points during the year. However, if these problems are presented as a unit, students cannot skip the hard problems and still do well on the unit. Teachers may choose to do all or just some of these questions. The goals are to incorporate calculus concepts early and often, to give students glimpses of future learning, and to show them how they will use this concept over the next four years.

Teacher Guide and Answer Key
Student Worksheet

[printer-friendly]

Teacher Guide, Geometry Activity 1

TEKS addressed: (G.7)(A); (G.8)(A); (G.8)(D); (G.11)(B); (G.11)(C)

This is a good Pre-AP* worksheet because it builds upon and extends skills that students have developed in middle school and Algebra I (using geometric formulas, solving literal equations, symbolic manipulation, function notation) while simultaneously addressing the TEKS listed above for high school geometry. It does so in a way very different from that seen in most textbooks. Most importantly, the context and wording is based upon an actual AP problem with which the students had difficulty.

1. Write an expression for the area of a circle as a function of its circumference.

$$A(C) = \frac{C^2}{4\pi}$$

2. Write an expression for the perimeter of a square as a function of its area.

$$P(A) = 4\sqrt{A}$$

3. If the volume of a right circular cylinder is 400π cubic units, write an expression for its lateral surface area as a function of its radius.

$$S(r) = \frac{800\pi}{r}$$

4. Water is being stored in a conical tank with height 12 feet and base diameter 8 feet. Write an expression for the volume of water in the conical tank as a function of h, the height of the water in the tank.

$$V(h) = \frac{1}{27}\pi h^3$$

5. Water is being stored in a cylindrical tank that has a base with area 900π square feet. Write an expression for the volume of water in the cylindrical tank as a function of h, the height of the water in the tank.

$$V(h) = 900\pi h$$

6. Write an expression for the area of an equilateral triangle as a function of the length of one of its sides.

$$A(s) = \frac{s^2\sqrt{3}}{4}$$

7. Write an expression for the area of a circle as a function of its diameter.

$$A(d) = \frac{\pi d^2}{4}$$

8. Write an expression for the surface area of a sphere as a function of its volume.

$$S(V) = 4\pi\left(\frac{3V}{4\pi}\right)^{\frac{2}{3}}$$

9. Write an expression for the volume of a cube as a function of its surface area.

$$V(S) = \left(\frac{S}{6}\right)^{\frac{3}{2}}$$

10. If the area of the base of a cone is 25π square units, express the volume of the cone as a function of its height.

$$V(h) = \frac{25\pi h}{3}$$

[printer-friendly]

For more information, please send an e-mail to Curriculum@tea.state.tx.us

Student Worksheet, Geometry Activity 1

1. Write an expression for the area of a circle as a function of its circumference.

2. Write an expression for the perimeter of a square as a function of its area.

3. If the volume of a right circular cylinder is 400π cubic units, write an expression for its lateral surface area as a function of its radius.

4. Water is being stored in a conical tank with height 12 feet and base diameter 8 feet. Write an expression for the volume of water in the conical tank as a function of h, the height of the water in the tank.

5. Water is being stored in a cylindrical tank that has a base with area 900π square feet. Write an expression for the volume of water in the cylindrical tank as a function of h, the height of the water in the tank.

6. Write an expression for the area of an equilateral triangle as a function of the length of one of its sides.

7. Write an expression for the area of a circle as a function of its diameter.

8. Write an expression for the surface area of a sphere as a function of its volume.

9. Write an expression for the volume of a cube as a function of its surface area.

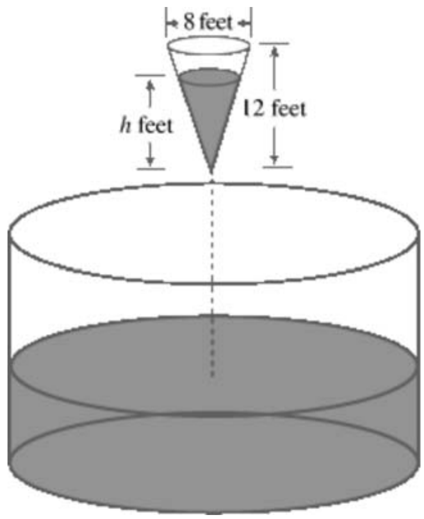
10. If the area of the base of a cone is 25π square units, express the volume of the cone as a function of its height.

[printer-friendly]

Teacher Guide, Geometry Activity 2

TEKS addressed: (a)(2); (a)(4); (a)(6); (G.1)(B); (G.5)(B)
Vocabulary includes literal equations and delta form of a slope equation.

This makes a good Pre-AP* geometry problem because students must apply many concepts in one problem. Teachers may choose to teach parts of this problem at different points during the year. However, if these problems are presented as a unit students can not skip the hard problems and still do well on the unit. Teachers may choose to do all or just some of these questions. The goal is to incorporate calculus concepts early and often, to give students glimpses of future learning, and to show them how they will use this concept over the next four years.



(a) List in words the items in the problem which DO NOT change.

The dimensions of the cone and the cylinder; the radius of the water in the cylinder

(b) List in words the items in the problem which DO change.

The height and radius of the water in the cone; the height of the water in the cylinder

(c) Write the formula for volume of a cone.

$$V = \frac{\pi}{3} r^2 h$$

(d) Find the capacity of the cone.

$$V = \frac{\pi}{3}(4^2)(12) = 64\pi$$

(e) Sketch a graph of height versus time as the water drains from the cone. Describe the slopes of the graph. Is the graph increasing, decreasing, concave up, concave down, etc.?

The water drains more slowly at the beginning, so the slopes are flatter. As the water drains, the slopes become steeper because the water drains faster. The graph is decreasing because the slopes are negative and concave down because the tangent lines would be above the curve.

(f) If the water is draining from the cone at a rate of 9 ft.3 per hour, how long would it take to empty the cone?

$$\frac{64\pi}{9\pi} = 7\ hrs.\ 6\frac{2}{3}\ \text{min.}$$

(g) Find the ratio of the radius of the water to the depth of the water in the cone.

1:3

(h) If the water in the cone is 6 foot deep, what is the radius of the water?

2 ft.

(i) If the water in the cone is 6 foot deep, what is the surface area of the water?

4π

(j) If the water in the cone is 6 foot deep, what is the volume of the water in the cone?

$\frac{\pi}{3}(2^2)(6) = 8\pi$ ft^3

(k) If the water in the cone is 6 foot deep, what is the ratio of the volume of the water in the cone to the capacity of the cone?

1:8

(l) If the water in the cone is 6 foot deep, what is the ratio of the radius of the water to the radius of the cone?

1:2

(m) What is the relationship between your answers for (h) and (i)?

Volume is the cube of the scale factor for the radius

(n) Find the radius of the water in the cone as a function of its height.

$$r = \frac{h}{3}$$

(o) Find the volume of the water in the cone as a function of the height.

$$V = \frac{\pi h^3}{27}$$

(p) Write a symbol to describe the change in height with respect to change in time using the delta form of a slope equation. Using a "d" for "delta" (Δ), rewrite the equation to represent an instantaneous rate of change.

$$\frac{\Delta V}{\Delta t} = \frac{dV}{dt}$$

(q) It can be determined that $\frac{dV}{dt} = \frac{\pi h^2}{9}\frac{dh}{dt}$ Write the meaning of these symbols in words.

The change in volume with respect to time = π / 9 times the height squared times the change in height with respect to time.

(r) The depth h, in feet, of the water in the cylindrical tank is changing at a rate of h-12 feet per minute. Write the statement in symbolic notation.

$$\frac{dh}{dt} = h - 12$$

(s) Rewrite the equation for the change in volume with respect to time using part (q).

$$\frac{dV}{dt} = \frac{\pi h^2}{9}(h - 12)$$

(t) At what rate is the volume of the water in the conical tank changing when h= 3? Write the sentence with symbols, then determine the rate. Indicate units.

Find $\frac{dV}{dt}$ when h = 3.

$$\frac{dV}{dt} = \frac{\pi(3)^2}{9}(3 - 12) = -9\pi \quad ft.^3/\text{min}$$

(u) What is the formula for volume of a cylinder using W for volume?

$W = \pi r^2 h$

(v) What is the volume of the cylinder in terms of y, the height of the water?

$W = 400\pi y$

(w) If the water in the cone is 6 foot deep when it begins to drain, what will be the height of the water in the cylinder when the cone is empty (assuming the cylinder starts off empty)?

$W = 400\pi h = 8\pi,\ so\ h = \frac{1}{50}$

(x) The change in the volume of the cylindrical tank can be described by the following:

$\frac{dW}{dt} = 400\pi \frac{dy}{dt}$ Write the meaning of the symbols in words, then solve algebraically for $\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{400\pi} \frac{dW}{dt}$

(y) Describe the difference in what is happening to the water in the conical tank and what is happening to the water in the cylindrical tank.

The water is draining from the conical tank, so the rate of change of volume is negative. The water is draining into the cylindrical tank, so the rate of change of volume will be the opposite of the rate of drainage.

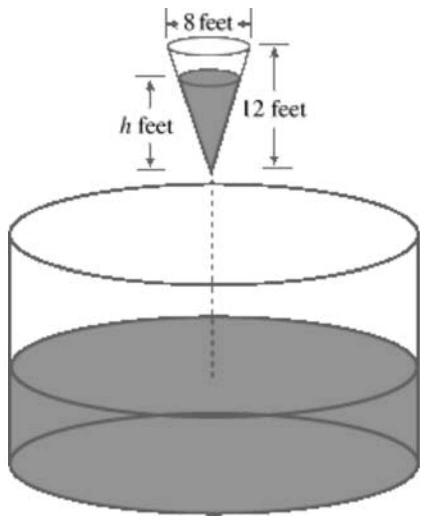
(z) At what rate is y changing when h=3? Write the sentence in symbolic notation, then solve indicating units of measure.

Find $\frac{dy}{dt}$ when h = 3. $\frac{dy}{dt} = \frac{1}{400\pi}(9\pi) = \frac{9}{400}\ ft / min$

[printer-friendly]

Student Worksheet, Geometry Activity 2

As shown in the figure below, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area of 400π square feet. The depth, h, in feet, of the water in the conical tank is changing at the rate of (h-12) feet per minute.



(a) List in words the items in the problem which DO NOT change.

(b) List in words the items in the problem which DO change.

(c) Write the formula for volume of a cone.

(d) Find the capacity of the cone.

(e) Sketch a graph of height versus time as the water drains from the cone. Describe the slopes of the graph. Is the graph increasing, decreasing, concave up, concave down, etc.?

(f) If the water is draining from the cone at a rate of 9 $ft.^3$ per hour, how long would it take to empty the cone?

(g) Find the ratio of the radius of the water to the depth of the water in the cone.

(h) If the water in the cone is 6 foot deep, what is the radius of the water?

(i) If the water in the cone is 6 foot deep, what is the surface area of the water?

(j) If the water in the cone is 6 foot deep, what is the volume of the water in the cone?

(k) If the water in the cone is 6 foot deep, what is the ratio of the volume of the water in the cone to the capacity of the cone?

(l) If the water in the cone is 6 foot deep, what is the ratio of the radius of the water to the radius of the cone?

(m) What is the relationship between your answers for (h) and (i)?

(n) Find the radius of the water in the cone as a function of its height.

(o) Find the volume of the water in the cone as a function of the height.

(p) Write a symbol to describe the change in height with respect to change in time using the delta form of a slope equation. Using a "d" for "delta" (Δ), rewrite the equation to represent an instantaneous rate of change.

(q) It can be determined that $\frac{dV}{dt} = \frac{\pi h^2}{9}\frac{dh}{dt}$ Write the meaning of these symbols in words.

(r) The depth h, in feet, of the water in the cylindrical tank is changing at a rate of h-12 feet per minute. Write the statement in symbolic notation.

(s) Rewrite the equation for the change in volume with respect to time using part (q).

(t) At what rate is the volume of the water in the conical tank changing when h= 3? Write the sentence with symbols, then determine the rate. Indicate units.

(u) What is the formula for volume of a cylinder using W for volume?

(v) What is the volume of the cylinder in terms of y, the height of the water?

(w) If the water in the cone is 6 foot deep when it begins to drain, what will be the height of the water in the cylinder when the cone is empty (assuming the cylinder starts off empty)?

(x) The change in the volume of the cylindrical tank can be described by the following:

$$\frac{dW}{dt} = 400\pi \frac{dy}{dt}$$

Write the meaning of the symbols in words, then solve algebraically for $\frac{dh}{dt}$

(y) Describe the difference in what is happening to the water in the conical tank and what is happening to the water in the cylindrical tank.

(z) At what rate is y changing when h=3? Write the sentence in symbolic notation, then solve indicating units of measure.

[printer-friendly]

Introduction to AP* Statistics Problem 6, 1997

On the AP* Statistics exam, the free-response section consists of five short-answer questions and one investigative task, a question that students usually spend about 30 minutes answering. The investigative task question from the 1997 AP Statistics exam follows. The 1997 question focuses on the concepts of analyzing data, finding models that correctly describe the data, writing equations for these models, and then interpreting the results.

When scoring the exam papers for this question, it became obvious that students had difficulty in working with data in contextual situations and seemed to have had little experience in exploring data that did not graph into a line. Many students did not seem to be familiar with the use of square roots and logarithms in data analysis. Although square roots and logarithms are introduced in Algebra II and further explored in Precalculus, it is usually not done in this context. Including this use of square roots and logarithms in the Pre-AP* classroom is a simple and worthwhile addition to the curriculum.

Many students upon seeing the data in this problem immediately entered it into their calculators and, only then, read the question. In this particular problem, all necessary computations had been given in the stem of the question. What students needed to do was to utilize the given models to determine an appropriate asking price for an automobile. Students in Pre-AP mathematics should have practice in producing various types of graphs, in determining appropriate mathematical models for data, and in utilizing a given model to make predictions. They should also know when a question is asking them to do one of the above. Finally, students had difficulty in coding data. They did not know that 1980, 1984, 1988, etc. could be written as 80, 84, and 88 and that the coded data was used in this problem.

Another major area of difficulty for students was being able to express their thoughts in coherent, concise sentences. This should be one of the goals of all Pre-AP mathematics courses. Many students could not explain their reasoning in part (d). Students also had difficulty in correctly interpreting the questions and therefore did not respond appropriately. In part (e), students seemed to think that the question was asking for their opinion about buying a car and did not understand that the prompt "Use some or all of the given data" meant that they had to base their answer strictly on the data and information given in the problem. Students were more likely to base their answer on personal experience.

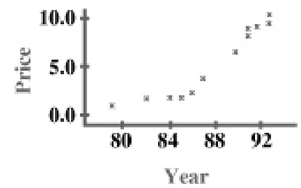
AP* Statistics Problem 6, 1997

You are planning to sell a used 1988 automobile and want to establish an asking price that is competitive with that of other cars of the same make and model that are on the market. A review of newspaper advertisements for used cars yields the following data for 12 different cars of this make and model. You want to fit a least square regression model to these data for use as a model in establishing the asking price for your car.

Production Year	1990	1991	1992	1987	1993	1991	1993	1985	1984	1982	1986	1979
Asking Price (in thousands of dollars)	6.0	7.7	8.8	3.4	9.8	8.4	8.9	1.5	1.6	1.4	2.0	1.0

The computer printouts for three different linear regression modles are shown below. Model 1 fits the asking price as a function of the production year, Model 2 fits the natural logarithm of the asking price as a function of the production year, and Model 3 fits the square root of the asking price as a function of the production year. Each printout also includes a plot of the residuals from the linear model *versus* the fitted values, as well as additional descriptive data produced from the least squares procedure.

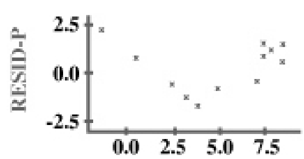
Model 1



The regression equation is Price = -58.1 + 0.179 Year.

Predictor	Coef	Stdev	t-ratio	p
Constant	-58.050	7.205	-8.06	0.000
Year	0.71900	0.08200	8.77	0.000

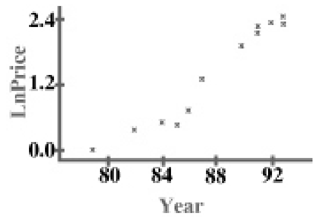
S= 0.1255 R-sq = 88.5%



Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	121.10	121.10	76.88	0.000
Error	10	15.75	1.58		
Total	11	136.85			

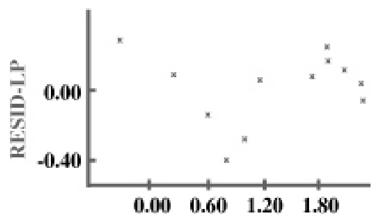
Model 2



The regression equation is LnPrice = -14.9 + 0.185 Year.

Predictor	Coef	Stdev	t-ratio	p
Constant	-14.924	1.223	-12.21	0.000
Year	0.18502	0.01392	13.30	0.000

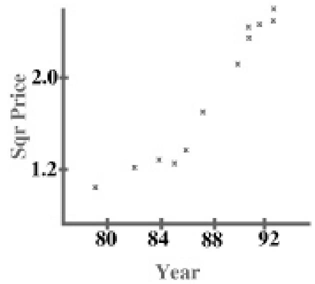
S= 0.2130 R-sq = 94.6%



Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	8.0190	8.0190	176.77	0.000
Error	10	0.4536	0.0454		
Total	11	8.4726			

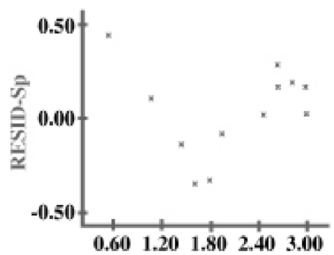
Model 3



The regression equation is Sqr = -13.3 + 0.176 Year.

Predictor	Coef	Stdev	t-ratio	p
Constant	-13.313	1.447	-9.20	0.000
Year	0.17559	0.01647	10.66	0.000

S= 0.2520 R-sq = 91.9%



Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	7.2221	7.2221	113.72	0.000
Error	10	0.6351	0.0635		
Total	11	7.8572			

1. Use Model 1 to establish an asking price for your 1988 automobile.
2. Use Model 2 to establish an asking price for your 1988 automobile.
3. Use Model 3 to establish an asking price for your 1988 automobile.
4. Describe any shortcomings you see in these three models.
5. Use some or all of the given data to find a better method for establishing an asking price for your 1988 automobile. Explain why your method is better.

[printer-friendly]

Middle School Activity Based on Problem 6

Based on AP* Statistics Problem 6, 1997

You are planning to sell a used 1988 automobile and want to establish an asking price that is competitive with that of other cars of the same make and model that are on the market. A review of newspaper advertisements for used cars yields the following data for 12 different cars of this make and model.

Production Year	1990	1991	1992	1987	1993	1991	1993	1985	1984	1982	1986	1979
Asking Price (in thousands of dollars)	6.0	7.7	8.8	3.4	9.8	8.4	8.9	1.5	1.6	1.4	2.0	1.0

6th Grade TEKS addressed: (6.1)(A); (6.1)(C); (6.2)(B); (6.10)(A); (6.10)(D); (6.11)(A); (6.11)(B); (6.12)(A); (6.12)(B)

A sixth grade activity based on this question with a sample of a student response is given on the next three pages.

7th Grade TEKS addressed: (7.11)(A); (7.11)(B); (7.12)(B)

Use this information to construct a scatter plot.

Estimate the asking price for your 1988 automobile. Give specific reasons how the information from the scatter plot helped you determine this price.

You are planning on buying a used 1995 automobile. Use the scatter plot to determine a price for this automobile. Justify your answer.

8th Grade TEKS addressed: (8.12)(A); (8.12)(B); (8.13)(B)

Use this information to construct a scatter plot. Find the median and the mean of the data in the chart.

Compare and contrast these three methods. Which method helps you determine the most reasonable asking price for your 1988 automobile? Give specific reasons why you think that one method is better than the other two.

Student Worksheet
Student Sample C

[printer-friendly]

Middle School Activity, Based on Problem 6

Based on AP* Statistics Problem 6, 1997

You are planning to sell a used 1988 automobile and want to establish an asking price that is competitive with that of other cars of the same make and model that are on the market. A review of newspaper advertisements for used cars yields the following data for 12 different cars of this make and model. Take this information and construct a graph with a meaningful scale and intervals. From your graph, estimate the cost of the 1988 automobile. Justify your answer. Estimate the cost of a 1995 automobile. Justify your answer.

Production Year	1990	1991	1992	1987	1993	1991	1993	1985	1984	1982	1986	1979
Asking Price (in thousands of dollars)	6.0	7.7	8.8	3.4	9.8	8.4	8.9	1.5	1.6	1.4	2.0	1.0

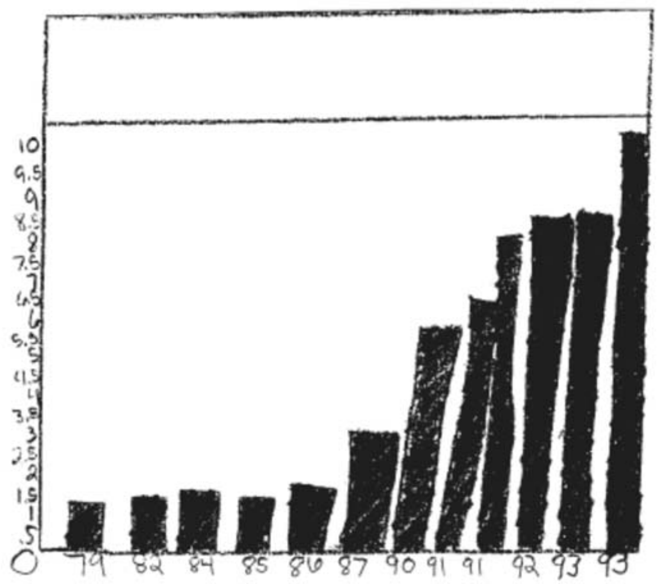
[printer-friendly]

Student Sample C

Statistics

You are planning to sell a used 1988 automobile and want to establish an asking price that is competitive with that of other cars of the same make and model that are on the market. A review of newspaper advertisements for used cards yields the following data for 12 different cars of this make and model. Take this information and construct a graph with a meaningful scale and intervals. From your graph, estimate the cost of the 1988 automobile. Justify your answer.

Production Year	**1990**	**1991**	**1992**	**1987**	**1993**	**1991**	**1993**	**1985**	**1984**	**1982**	**1986**	**1979**
Asking Price (in thousands of dollars)	6.0	7.7	8.8	3.4	9.8	8.4	8.9	1.5	1.6	1.4	2.0	1.0



Statistics

① How much would a 1988 automobile cost?

Our prediction was $4,700. We figured the question out by taking the cost of the 1990 automobile and the 1987 automobile and subtracted them. We got the answer of $2,600 and divided by the 2 years that were between 1900 and 1987. Then we took the answer $1,300 and added it to the 1987 automobile $3,400. That's how we got the answer of $4,700.

② How much would a 1995 automobile cost?

Our prediction was $11,700. We figured the problem out by subtracting the 1990 cost from the 1993 cost. We got the answer $3,800 and divided by 4 (1900 both 1991, 1992, and both 1993) to get the answer of $950. We timesed $950 by 2 the years 1994 and 1995. Then we added the cost of the 1993 automobile to our answer 1,900 and got the answer of $11,700.

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Algebra I Activity

Based on AP* Statistics Problem 6, 1997

Student Worksheet
Teacher Guide and Answer Key

[printer-friendly]

For more information, please send an e-mail to Curriculum@tea.state.tx.us

Student Worksheet, Algebra I Activity

Based on AP* Statistics Problem 6, 1997

Often one looks at data to formulate a picture or make a decision about a past, present, or future event.

It is important to remember that when answering questions about real data that your answers are written in complete sentences and in the context of the problem.

You are planning to sell a used 1988 automobile and want to establish an asking price that is competitive with that of other cars of the same make and model that are on the market. A review of newspaper advertisements for used cars yields the following data for 12 different cars of this make and model.

Production Year	1990	1991	1992	1987	1993	1991	1993	1985	1984	1982	1986	1979
Asking Price in $	6000	7700	8800	3400	9800	8400	8900	1500	1600	1400	2000	1000

1. Often it is difficult to work with large or small data, for example numbers as large as $2,300,000,000. It is easier to work with coded data, that is, data that has been recalculated by a rule or transformation. In the above case, we would rewrite the number as $2.3 billion.

a. Use this idea to code the data in the table above. For the data, Production Year, calculate the number of years since 1979. Using this rule, 1979 will be coded as 0 and 1980 will be coded as 1, and so forth.

b. Code the asking price of the automobile in thousands of dollars. For example, $7700 will be written as $7.7 (thousands of dollars).

2. Complete the table of values below using your coded data.

Production Year	Asking Price (in thousands of $)
(1990) 11	6
(1991) 12	7.7
...	...

3. Draw a scatter plot of the coded data. Be sure to label your axes with the appropriate variables and scales.

4. Write an equation for a linear model for this data.

a. What is the slope of the line? In terms of the problem, explain the meaning of the slope.

b. What is the Asking Price intercept (y-intercept)? In terms of the problem, explain the meaning of the Asking Price intercept (y-intercept).

5. Use your model to predict a selling price for your 1988 automobile.

a. Do you think that this price is a reasonable answer for this automobile? Use the data in the problem to support your answer. (Remember to use complete sentences.)

b. What is the Asking Price intercept (y-intercept)? In terms of the problem, explain the meaning of the Asking Price intercept (y-intercept).

6. A friend bought a new car last year, and he decides to sell it now. Use your model to predict the price of this car. Do you think that this is a reasonable price for this automobile? Support your answer. (Remember to use complete sentences.)

7. In questions 5 and 6, you used your mathematical model to predict the asking price of two automobiles. Explain which of the two prices you would expect to be the most accurate and why.

8. Using the coded data and the linear model you wrote in problem 4, complete the table of values below.

Year	Actual Asking Price (in thousands of $)	Predicted Asking Price (in thousands of $)	Difference (Actual - Predicted)
...	...	...	...

9. Draw a scatter plot for (year, difference).

10. If all of the Predicted Asking Price values were equal to the Actual Asking Price values, where would the points (year, difference) be located on the graph in problem 9? What would this tell you about your model?

[printer-friendly]

Teacher Guide and Answer Key, Algebra 1 Activity

Based on AP* Statistics Problem 6, 1997

Algebra I TEKS addressed: (a)(3); (a)(4); (a)(5); (a)(6); (A.1)(B); (A.1)(C); (A.1)(D); (A.1)(E); (A.2)(B); (A.2)(C); (A.2)(D); (A.3)(A); (A.5)(A); (A.5)(C); (A.6)(B); (A.6)(D); (A.7)(B); (A.7)(C)

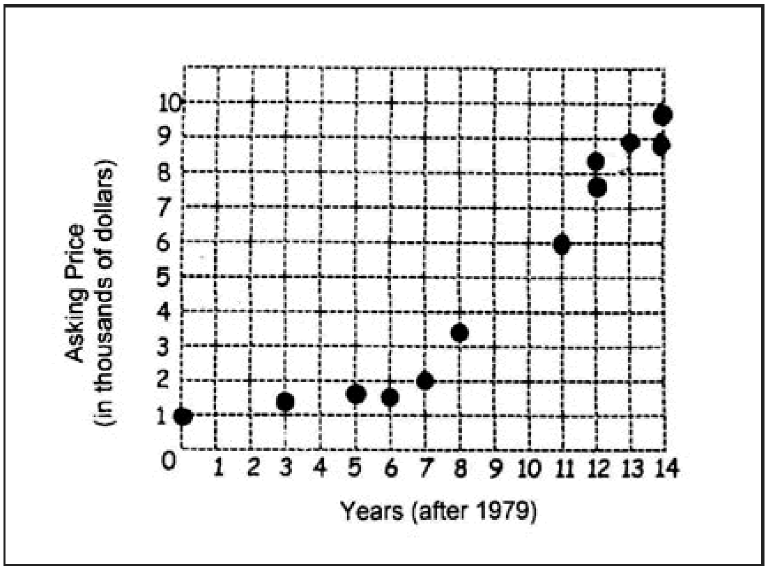
This problem is an example of a good Pre-AP* problem because it requires students to incorporate many concepts into one problem. Students are asked to interpret equations (linear) and graphs and explain the reasons for their interpretations. The problem begins as a typical Algebra 1 scatter plot problem (in questions 1-4). Questions 5-10 extend the problem to concepts the students will encounter in an AP Statistics class. Students are asked to evaluate the linear model they found for the given data.

1. a. & b. See the table of values below for problem 2

2.

Production Year	Asking Price (in thousands of $)
(1990) 11	6
(1991) 12	7.7
(1992) 13	8.8
8	3.4
14	9.8
12	8.4
14	8.9
6	1.5
5	1.6
3	1.4
7	2
0	1

3.



4.
a. We will use y = .5x + 1. *(Answers will vary because the students are not using a graphing calculator to find the equations. The students will use their knowledge of linear functions and the graphs of linear functions to write a linear function that models this data. They will use their model to help them answer the remaining questions.)*

What does the slope represent?

- the predicted change in the asking price (in thousands of $) divided by the change in the years (after 1979); or
- a predicted change of $500 per year; or
- for every year after 1979, an expected increase in asking price of 0.5 thousands of dollars.

b. The asking price-intercept is 1. This represents the asking price ($1000) for the year 0 (1979).

5. Using the model written in #4, the asking price might be $5500. When I plotted the price (9, 5.5) on my scatter plot, the price appears too high because year 8 is $3400 and year 11 is $6000. I think the price should be closer to year 8 than year 11.

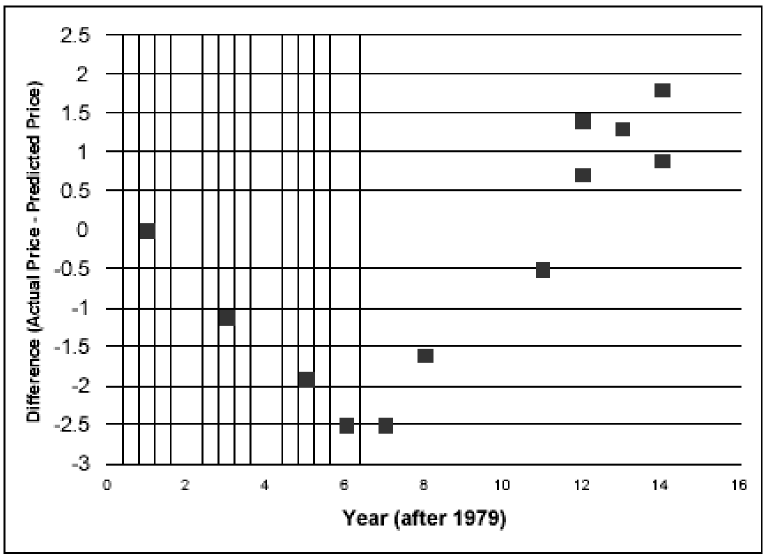
6. For this example we will let the year 2002 represent the present year so that using the model written in #4 the asking price might be $12,500. When I extended my graph to include this data, this price appears to be too low. If I use the rate of change that I found in #4a, then from year 14 to year 23 the price should have increased about $4500. Therefore, the predicted price in 2002 would be $8900 + $4500 = $13,400.

7. Answers to this question will vary. Teachers should discuss that question #5 is an example of interpolation (predicting within the range of the given data set) and question #6 is an example of extrapolation (predicting outside of the range of the given data set). Interpolation is more reliable than extrapolation.

8.

Year	Actual Asking Price (in thousands of $)	Predicted Asking Price (in thousands of $)	Difference Actual - Predicted
0	1	1	0
3	1.4	2.5	-1.1
5	1.6	3.5	-1.9
6	1.5	4	-2.5
7	2	4.5	-2.5
8	3.4	5	-1.6
11	6	6.5	-0.5
12	7.7	7	0.7
12	8.4	7	1.4
13	8.8	7.5	1.3
14	9.8	8	1.8
14	8.9	8	0.9

9.



10. The points (year, difference) would lie on the line y = 0 (difference = 0). This would indicate that this model was a good model since the difference between the actual and predicted was 0, meaning that the predicted price was the same as the actual price. On my graph the early year prices are negative, indicating that the predicted price is more than the actual price and the later year prices are positive, indicating that the predicted price is less than the actual price. This graph would help support my answers to problems #5 and #6.

[printer-friendly]

For more information, please send an e-mail to Curriculum@tea.state.tx.us

Algebra II Activity

Based on AP* Statistics Problem 6, 1997

Student Worksheet
Teacher Guide and Answer Key

[printer-friendly]

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Student Worksheet, Algebra II Activity

Based on AP* Statistics Problem 6, 1997

You are planning to sell a used 1988 automobile and want to establish an asking price that is competitive with that of other cars of the same make and model that are on the market. A review of newspaper advertisements for used cars yields the following data for 12 different cars of this make and model.

Production Year	1990	1991	1992	1987	1993	1991	1993	1985	1984	1982	1986	1979
Asking Price in $	6000	7700	8800	3400	9800	8400	8900	1500	1600	1400	2000	1000

1. Draw a scatter plot of the data. Remember to scale and label both axes.

2. Because of the apparent curvature of the graph, you decide that an exponential function would be a better model for this data. Write an exponential model for this data. Does the exponential function seem to be a good model for the data? Explain why or why not.

3. Use the exponential function to determine a selling price for your 1988 automobile.

4. Your friend thinks that the data would be better represented by using two line segments. Write a piecewise function for the data using two lines or segments of lines.

5. Use your piecewise defined function to determine a selling price for your 1988 automobile.

6. A friend bought a new car last year and he decides to sell it now. Use your exponential model and your piecewise model to predict the price of this car. Which of the two prices would be the most accurate and why?

7. Based on your answer to questions 2-6, decide which model, exponential or piecewise, is a better mathematical model for this situation. Explain why you chose this model.

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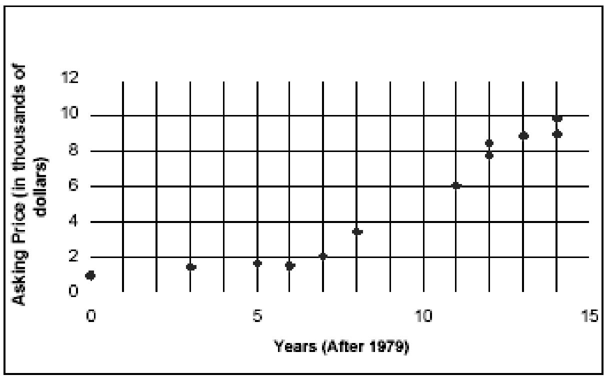
Teacher Guide and Answer Key, Algebra II Activity

Based on AP* Statistics Problem 6, 1997

Algebra II TEKS addressed: (a)(3);(a)(5); (2A.1)(A); (2A.1)(B); (2A.11)(F)

This problem is a good Pre-AP* problem because it requires students to work with exponential and linear functions in a problem-solving situation. Students are expected to use their knowledge of these functions to write a mathematical model for the given data. The data can be plotted on a graphing calculator. A student can see a graph of his/her model on the calculator then adjust his/her model as necessary. This problem builds a greater understanding of transformations of functions. This problem also introduces a piecewise function. Although piecewise functions are not addressed in the TEKS until Precalculus, piecewise functions can be written and interpreted in an Algebra II Pre-AP class using pieces of the functions that are studied in the Algebra II TEKS.

1.



2. y = (1.17)x The asking price using this exponential function is too low for 5 = years = 7 and too high for 12 = years = 14.

3. The asking price for year 9 would be about $4108.40 if you use the exponential model.

4.

$$y = \begin{cases} .1x & \text{for } 0 \le x \le 7 \\ 1.15(x-7)+2 & \text{for } x \ge 7 \end{cases}$$

5. The asking price for year 9 would be about $4300 if you use the piecewise model.

6. Using the exponential model for the year 23, the asking price would be about $37,006.22. Using the piecewise model for the year 23, the asking price would be about $20,400. The piecewise model would be more appropriate because the exponential model expected asking price is too high for the years between 12 and 14 thus I think it would be too high for any year after the 14th.

7. I think the piecewise function better models the data because values of the data are closer to the piecewise function than to the exponential functions. Also, the rate of change in the exponential function continues to increase which does not seem to fit the context for the asking price of the automobiles. It appears from the scatter plot that the rate of change for the asking price of automobiles is a constant value and therefore linear.

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Precalculus Activity

Based on AP* Statistics Problem 6, 1997

Student Worksheet
Teacher Guide and Answer Key

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Student Worksheet, Precalculus Activity

Based on AP* Statistics Problem 6, 1997

You are planning to sell a used 1988 automobile and want to establish an asking price that is competitive with that of other cars of the same make and model that are on the market. A review of newspaper advertisements for used cars yields the following data for 12 different cars of this make and model. You want to fit a least squares regression model to these data for use as a model in establishing the asking price for your car.

Production Year	1990	1991	1992	1987	1993	1991	1993	1985	1984	1982	1986	1979
Asking Price (in thousands of dollars)	6.0	7.7	8.8	3.4	9.8	8.4	8.9	1.5	1.6	1.4	2.0	1.0

Establish an asking price for the 1988 automobile using 4 models (linear, exponential, quadratic, and power). Let t = years since 1978 and p = price in thousands of dollars. Use your calculator's linear regression feature to determine a linear function in the form of:

a) $p = mt + b$ using (t,p) data.

b) $\ln p = mt + b$ using the transformed data (t, ln p). Using exponential/logarithmic properties, re-express the equation as an exponential function in the form of $p = ae^{kt}$ to determine the asking price of the automobile.

c) $\text{sqrt}(p) = mt + b$ using the transformed data (t, sqrt(p)). Using algebraic properties, re-express the equation in quadratic form, $p = at^2 + bt + c$, to determine the asking price of the automobile.

d) $\ln p = m \cdot \ln t + b$ using the transformed data (ln t, ln p). Using exponential/logarithmic properties, re-express the equation as a power function, $p = at^b$ to determine the asking price of the automobile.

e) Graph the 4 models with the data points. Which appears to be the best model? Explain why.

[printer-friendly]

Teacher Guide and Answer Key, Precalculus Activity

Based on AP* Statistics Problem 6, 1997

TEKS addressed: (P.3)(B); (P.3)(C); (P.3)(D)

You are planning to sell a used 1988 automobile and want to establish an asking price that is competitive with that of other cars of the same make and model that are on the market. A review of newspaper advertisements for used cars yields the following data for 12 different cars of this make and model. You want to fit a least squares regression model to these data for use as a model in establishing the asking price for your car.

Production Year	1990	1991	1992	1987	1993	1991	1993	1985	1984	1982	1986	1979
Asking Price (in thousands of dollars)	6.0	7.7	8.8	3.4	9.8	8.4	8.9	1.5	1.6	1.4	2.0	1.0

Establish an asking price for the 1988 automobile using 4 models (linear, exponential, quadratic, and power). Let t = years since 1978 and p = price in thousands of dollars. Use your calculator's linear regression feature to determine a linear function in the form of:

a) $p = mt + \text{b}$ using (t,p) data. **$p = .718997t - 1.968552$**

$p(10) = 5.2$ Therefore, using the linear model, the predicted asking price for a 1988 automobile is $5200.

b) ln p = mt + b using the transformed data (t, ln p). Using exponential/logarithmic properties, re-express the equation as an exponential function in the form of $p = ae^{kt}$ to determine the asking price.

$\ln p = .185021t - .492845$

$e^{\ln p} = e^{.185021t - .492845}$

$p = e^{.185021t} \div e^{.492845}$

$p = .610886\, e^{.185021t}$

$p(10) = 3.9$ Therefore, using this exponential model, the predicted asking price for a 1988 automobile would be $3900.

c) sqrt(p) = mt + b using the transformed data (t, sqrt(p)). Using algebraic properties, re-express the equation in quadratic form, $p = at^2 + bt + c$, to determine the asking price of the automobile.

$\text{sqrt}(p) = .175587t + .382519$

$\text{sqrt}(p)^2 = (.175587t + .382519)^2$

$p = .030831t^2 + .134331t + .146321$

p(10) = 4.6 Therefore, using this quadratic model, the predicted asking price for a 1988 automobile would be $4600.

d) ln p = m · ln t + b using the transformed data (ln t, ln p). Using exponential/logarithmic properties, re-express the equation as a power function, $p = at^b$, to determine the asking price of the automobile.

ln *p* = .953034 ln t - .679989

$e^{\ln p} = e^{.95034 \ln t - .679989}$

$p = e^{.95034 \ln t} \div e^{.679989}$

$p = 0.506623 \bullet (t^{.953034})$

***p*(10) = 4.5 Therefore, using this power model, the predicted asking price of a 1988 automobile would be $4500.**

e) Graph the 4 models with the data points. Which appears to be the best model? Explain why.

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What Makes a Good Pre-AP* Mathematics Problem?

After reading the goals of the Pre-AP* mathematics program, seeing how AP* problems can be adapted for use at other levels. After examining exemplar problems for particular TEKS, we hope you have a better understanding of what makes a problem or activity particularly appropriate for use in the Pre-AP mathematics classroom. Below you will find some of the criteria that the committee used in selecting the problems and activities for this document.

A good Pre-AP mathematics problem or activity

- has a clear connection to the vocabulary, skills, concepts, or habits of mind necessary for success in AP mathematics courses;
- goes beyond a minimalist approach to addressing the TEKS;
- can serve multiple purposes, such as addressing an Algebra I TEKS, reviewing a middle school geometry skill, and introducing an AP Calculus concept;
- should go beyond simple drill and recall (There should be a greater emphasis on analysis, application, and synthesis of material.);
- requires students to engage in an extended chain of reasoning (Problems should require more than one step and might cover more than one topic.);
- might be completely different from problems that the teacher has demonstrated in class, though based on the same concept (Students are expected to apply their knowledge in novel situations with very little teacher direction.);
- requires students to develop their reading and interpretation skills using verbal, graphical, analytical, and numerical prompts;
- asks students to communicate their thoughts orally and/or in writing (Students must be able to justify their work in clear, concise, and well-written sentences.);
- stretches the students in ways that might make them uncomfortable (The solving of problems might take several attempts. They might have to hear someone else's explanation (preferably one of their peers) before they begin to develop understanding.);
- should be graded based on the process and methods as well as the final answers; and
- might require the thoughtful use of technology.

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Where To Now

The Mathematics Lighthouse Initiative is meant to provide you with some guidance as you navigate past the rocky shoals of beginning an effective and inclusive Pre-AP* mathematics program. It is by no means meant to provide you with a specific curriculum or complete set of activities for a Pre-AP class. We hope that what you have read on this Web site will cause you to examine your current practice, give you some new ideas, and inspire you to seek further information.

Now you are left to head out into the open seas where exciting adventures, turbulent waters, and untold riches await you. We hope that you are not making this voyage alone. The best Pre-AP mathematics programs are built by teachers working together in vertical teams. We hope that you also will utilize the experiences of others who have made this voyage before you. Many of these earlier voyagers have left behind maps and other guidance in the form of curriculum documents, publications, problem sets, etc. Ultimately, your voyage will be unique, of course, tailored to the particular needs of your district, your students, and your teaching style.

You can connect with other voyagers by attending conferences (particularly those sponsored by the College Board), summer institutes, and other training opportunities.

Good luck on your journey. We hope the winds of change will fill your sails and bring you and your students to new and interesting ports of call.

The world is round and the place that may seem like the end may also be only the beginning.
–Ivy Baker Priest

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Texas Essential Knowledge and Skills (TEKS) Charts

Click on a grade level or content area below to view the TEKS chart with suggested activities for that area:

- Grade 6
- Grade 7
- Grade 8
- Algebra I
- Algebra II
- Geometry
- Precalculus

Introduction

As the Lighthouse committee began its examination of the Texas Essential Knowledge and Skills (TEKS), we were often surprised by what was included or left out of courses that preceded or followed those that we normally teach.

"Do they really expect eighth graders to be able to do that?"

"Where are the sequences and series that we used to do in Algebra II?"

Ultimately, we agreed that all of the concepts and skills necessary to prepare students for success in AP* Statistics and AP Calculus would be covered if the TEKS were interpreted in a particular way. Due to time constraints, we were reluctant to add any additional topics to the TEKS, though a teacher might choose to do so.

The problem is particularly acute at the middle school level when all of the TEKS for grades 6-8 are often covered in only two years in order for students to take Algebra I in grade 8. Having students just skip over a year of elementary or middle school mathematics is a dangerous proposition that can have serious repercussions in subsequent courses. A well-planned and instructed Pre-AP* middle school program combines, streamlines, and collapses the material in such a way that all of the TEKS are addressed at a deeper and more complex level.

At one point, someone on the committee said, "The problem is not that the TEKS are incomplete; it is that all of these things are treated equally. Some of these TEKS are three-minute topics, and some of them are three-week topics." That gave us our idea for the structure of the charts in this section. We went through the TEKS and sorted them into three groups.

- The TEKS in regular font are topics with which students already have some familiarity due to previous instruction and which are being revisited through the spiraling curriculum or are topics that can be covered in minimal time. These topics might provide foundational knowledge (such as definitions) that will be used for future topics throughout the course.

- *The TEKS typed in italics are topics that might be addressed throughout the course on multiple occasions or might be addressed to greater depth than the previous topics.*

- **The TEKS in a bold, slightly larger, font are those that merit greater time commitment and greater depth of understanding for the Pre-AP student. These topics should be taught with a particular emphasis toward preparing students for AP Calculus or AP Statistics.**

After categorizing the TEKS, we looked for problems or activities that would exemplify those TEKS in the third group and included them in the second column as examples of what we felt were good Pre-AP mathematics problems and activities. Remember that these are only examples; students will have to do many more than the few problems that we were able to include here in order to be well-prepared for AP Statistics and AP Calculus. These are meant to give you ideas and get you started in understanding what makes a good Pre-AP mathematics problem. You will also find in the second column additional comments about the TEKS or sample problems that we felt might be important.

Click on a grade level or area below to view the TEKS chart with suggested activities for that area.

- Grade 6
- Grade 7
- Grade 8
- Algebra I
- Algebra II
- Geometry
- Precalculus

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For more information, please send an e-mail to Curriculum@tea.state.tx.us

TEKS: Grade 6

Read an introduction to Texas Essential Knowledge and Skills charts.

TEKS	Examples	Commentary
111.22. MATHEMATICS, GRADE 6 6.1 Number, operation, and quantitative reasoning. The student represents and uses rational numbers in a variety of equivalent forms. (A) compare and order non-negative rational numbers; *(B) generate equivalent forms of rational numbers including whole numbers, fractions, and decimals;* *(C) use integers to represent real life situations;* *(D) write prime factorizations using exponents;* *(E) identify factors of a positive integer, common factors, and the greatest common factor of a set of positive integers; and* *(F) identify multiples of a positive integer and common multiples and the least common multiple of a set of positive integers.*		
6.2 Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve problems and justify solutions. (A) model addition and subtraction situations involving fractions with		

objects, pictures, words, and numbers; *(B) use addition and subtraction to solve problems involving fractions and decimals;*		
(C) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates;	Measure with string around a globe at the equator and call the length e. Place this string on a latitude line on the globe between the cities of Ahmadabad, India, and Cabo San Lucas, Mexico; call this distance d. Measure e and d in centimeters. Set up a proportion to find the actual distance in kilometers between the two cities using the following proportion: d (in cm) / e (in cm) ? (km) / 40,000km	Extension: Have the students use an atlas to find the actual distance between the two cities. Estimate distances between other cities. Use different size globes and determine if the size of the globe makes a difference in distances.
(D) estimate and round to approximate reasonable results and to solve problems where exact answers are not required; and *(E) use order of operations to simplify whole number expressions (without exponents) in problem solving situations.*		
6.3 Patterns, relationships, and algebraic thinking. The student solves problems involving direct proportional relationships.		
(A) use ratios to describe proportional situations; **(B) represent ratios and percents with concrete models, fractions, and decimals; and**	Use a small bag of M&Ms to make inferences about the proportion of certain colors of M&Ms in a larger bag. Collect, organize, and analyze data; use ratios to compare quantities of different colors; use equivalent fractions to understand proportions; and make predictions from data using proportions. Have students make a chart showing the number of each color, the corresponding fraction, decimal, and percent. Draw a bar graph using the data in percent form. Compare among students or groups.	Compare with M&M/Mars Co published proportions: Red 20%, green 10%, yellow 20%, orange 10%, brown 30%, blue 10%

	Use a calculator to find out the number of degrees in a circle represented by the percent of M&Ms by color. Use this information to make a circle graph.	
(C) use ratios to make predictions in proportional situations.	Collect data in class such as how many students wear glasses. Based on the data, make a prediction for the entire grade level. Have several students count at lunch to determine the accuracy of the prediction.	Extension: Discuss if the class is an unbiased sample of the grade level. Talk about different methods of sampling and ways of eliminating bias in sampling.
6.4 Patterns, relationships, and algebraic thinking. The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes.		
(A) use tables and symbols to represent and describe proportional and other relationships such as those involving conversions, arithmetic sequences (with a constant rate of change), perimeter, and area; and	Arrange 12 square tiles to form a rectangle. What is the length and width of your rectangle? a. Use tiles to form as many rectangles as you can with an area of 12 square tiles. Use this information to complete a table of values. b. Graph the data pairs (length, width) from your table on a coordinate plane. c. As the length of the rectangle gets larger, describe what happens to the width. d. Can a rectangle ever have a width of 0 units? Justify your answer. e. As the length of the rectangle gets smaller, describe what happens to the width. f. Can a rectangle ever have a length of 0 units? Justify your answer. g. Write an equation relating the length and width. Solve your equation for width. Explain how your equation supports your answer to part d and f.	

(B) use tables of data to generate formulas representing relationships involving perimeter, area, volume of a rectangular prism, etc.	From a piece of paper that is 8.5 inches by 11 inches, cut a one-inch square out of each corner. Fold the paper into a box without a lid. What is the volume of the box? What would happen to the volume if you cut a two-inch square out of each corner instead of one-inch? What if you cut 3 inches? If you were designing a box using this piece of paper that would hold the most popcorn, what would be the dimensions of this box? Use a table and graph to help explain your answer. What would be a reasonable domain in this problem? Write an equation that would give the volume of the box for any given cutout square.	This type of problem leads to a calculus topic, optimization. A similar problem is in the NCTM Addenda Series, Grades 5-8: Patterns and Functions, pages 64-65
6.5 Patterns, relationships, and algebraic thinking. The student uses letters to represent an unknown in an equation. **formulate equations from problem situations described by linear relationships.** 6.6 Geometry and spatial reasoning. The student uses geometric vocabulary to describe angles, polygons, and circles. (A) use angle measurements to classify angles as acute, obtuse, or right; (B) identify relationships involving angles in triangles and quadrilaterals; and (C) describe the relationship between radius diameter and circumference of a circle.	Samuel checked out the inside of the family refrigerator. He saw 4 times as many apples as strawberries and half as many oranges as apples. Represent this as many ways as you can with a diagram, table, and number sentence.	
6.7 Geometry and spatial reasoning. The student uses coordinate geometry to identify a location in 2 dimensions.		

locate and name points on a coordinate plane using ordered pairs of non-negative rational numbers. 6.8 Measurement. The student solves application problems involving estimation and measurement of length, area, time, temperature, volume, weight, and angles. (A) estimate measurements (including circumference) and evaluate reasonableness of results;		
(B) select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter), area, time, temperature, volume, and weight;	Daniel collects ducks and dachshunds. This morning he counted 20 heads and 56 legs. How many ducks and dachshunds does he have? Daniel plans to build a pet house to keep them in. He plans to follow the guidelines of the local animal shelter. They say for each dog the cage must be at least 27 cubic feet and for 3 ducks, the cage must be at least 16 cubic feet. If the pet house is 9 feet wide, 20 feet long, with an 8 feet ceiling, will he have enough space for all his pets?	
(C) measure angles; and *(D) convert measures within the same measurement system (customary and metric) based on relationships between units.*		
6.9 Probability and statistics. The student uses experimental and theoretical probability to make predictions. *(A) construct sample spaces using lists and tree diagrams; and* *(B) find the probabilities*		

of a simple event and its complement and describe the relationship between the two.		
6.10 Probability and statistics. The student uses statistical representations to analyze data.		
(A) select and use an appropriate representation for presenting and displaying different graphical representations of the same data including line plot, line graph, bar graph, and stem and leaf plot;	Collect data of interest to the class. Use this data to make box plots, stem and leaf, scatter plots and number line plots. Examples of data: • Number of hours spent on homework per week; • Number of hours spent watching TV per week; • Prices of favorite clothing item; and • Home run leaders of the American and National Leagues from 1950-2001.	Ideas from *Exploring Data* (1996); Dale Seymour Publications. This introduces the AP* Statistics concept of comparing distributions of univariate data-dot plots, back-to-back stemplots, and parallel boxplots.
(B) identify mean (using concrete objects and pictorial models), median, mode, and range of a set of data; *(C) sketch circle graphs to display data; and* ***(D) solve problems by collecting, organizing, displaying, and interpreting data.***	The hare challenged the tortoise to several 100 meter races. In the first race, the tortoise completed the race in 25 minutes. Give an equation that solves for rate and graph his progress on a grid. The hare left 20 minutes after the tortoise and raced at a speed of 20 meters per minute. Graph his progress and decide who won the race. In the second race, the tortoise traveled 5 meters per minute. The hare left 5 minutes after the tortoise and ran for 2 minutes, stopped for 14 minutes, then realized he was behind, and he continued to race. Did he overtake the hare? At what point? Draw a chart and graph of the race.	This problem introduces the concepts of rate of change and systems of equations. It is also an example of a piecewise function.

6.11 Underlying processes and mathematical tools. The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities inside and outside of school.		The underlying processes and mathematical tools are the same for grades 6-8. TEKS 7.13, 7.14, 7.15 and 8.14, 8.15, 8.16 are the same as 6.11, 6.12, and 6.13. Although the TEKS are the same, older students should show greater sophistication in their mathematical reasoning and communication.
(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematics topics; *(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;* *(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and* *(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.*		
6.12 Underlying processes and mathematical tools. The student communicates about Grade 6		

mathematics through informal and mathematical language, representations, and models. *(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and* (B) evaluate the effectiveness of different representations to communicate ideas.		
6.13 Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. *(A) make conjectures from patterns or sets of examples and non-examples; and* *(B) validate his/her conclusions using mathematical properties and relationships.*		

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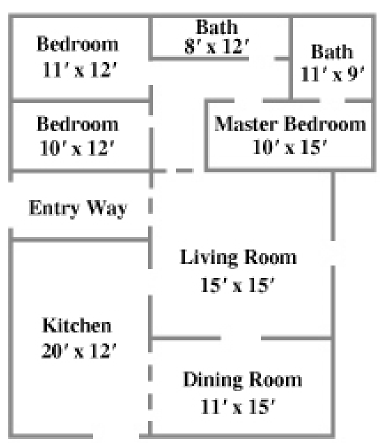
TEKS: Grade 7

Read an introduction to Texas Essential Knowledge and Skills charts.

TEKS	Examples	Commentary
111.23. MATHEMATICS, GRADE 7. 7.1 Number, operation, and quantitative reasoning. The student represents and uses numbers in a variety of equivalent forms. (A) compare and order integers and positive rational numbers; (B) convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator; and (C) represent squares and square roots using geometric models.		
7.2 Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. (A) represent multiplication and division situations involving fractions and decimals with models including concrete objects, pictures, words,		

and numbers; *(B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals;* *(C) use models such as concrete objects, pictorial models, and number lines, to add, subtract, multiply, and divide integers and connect the actions to algorithms;*		
(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio;	Using a map or road atlas, choose two cities that are at least 700 miles apart. Determine the distance between them. Determine how much time that you would save driving from one city to another at 70 miles per hour instead of 55 mph. If your car averages 27 miles per gallon when traveling 55 mph, determine the cost of the trip at this speed. If your car averages 22 miles per gallon when traveling at 70 mph, determine the cost of the trip at this speed. Compare the two costs. Using this data, write a paragraph describing which speed is best in terms of time, cost. The diameter of the earth is about 8,000 miles. What would your speed be in miles per hour if you took 80 days to circumnavigate the earth at the equator?	A variation of this problem would be to choose two cities that are relatively close together, 40 miles or less. This activity was adapted from *Real Life Math: Algebra, Walch Education,* page 188.
(E) simplify numerical expressions involving order of operations and exponents; *(F) select and use appropriate operations to solve problems and justify the selections; and*		

(G) determine the reasonableness of a solution to a problem.		
7.3 Patterns, relationships and algebraic thinking. The student solves problems involving direct proportional relationships. *(A) estimate and find solutions to application problems involving percent; and*		
(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.	You are building your dream home. It is time now to determine the costs of your flooring. You have found a carpet for the bedrooms and the hallways. Because of the amount of foot traffic in the kitchen, entryway, and bathrooms, you have chosen a ceramic tile. In the living room and dining room, you want a hardwood floor. Using these costs, determine the total cost of flooring for your home. Carpet costs $20.95 per sq. yard. Tile, $5.50 per sq foot. Hardwood flooring is $9.00 per sq foot.  	
7.4 Patterns, relationships, and algebraic thinking. The student represents a		

relationship in numerical, geometric, verbal and symbolic form. **(A) generate formulas involving conversions, perimeter, area, circumference, volume, and scaling;**	1. Distribute to your students circular lids, cans and other items of various sizes. Have the students measure the circumference and diameter of these items. Record the data in a table. Determine the relationship between the diameter and the circumference. Graph the data using the diameter as the independent variable. What is the slope of the line? Write an equation that expresses the relationship between the diameter and the circumference. 2. Roll a 5-inch by 8-inch index card into a cylinder by taping the shorter sides together. Roll another 5 x 8 index card into a cylinder by taping the long sides together. Use rice to compare their volumes. Does one hold more than the other? What is the volume of each? Write a summary sentence that explains what this activity shows.	NCTM Addenda Series, Grades 5-8, discusses this problem (*Patterns and Functions* and *Measurement in the Middle Grades)*.
(B) graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling; and *(C) use words and symbols to describe the relationship between the terms in an arithmetic sequence (with a constant rate of change) and their positions in the sequence.*		
7.5 Patterns, relationships, and algebraic thinking. The student uses equations to solve		

problems. (A) use concrete and pictorial models to solve equations and use symbols to record the actions; and *(B) formulate problem situations when given a simple equation and formulate an equation when given a problem situation.* 7.6 Geometry and spatial reasoning. The student compares and classifies two- and three-dimensional figures using geometric vocabulary and properties. (A) use angle measurements to classify pairs of angles as complementary or supplementary; (B) use properties to classify triangles and quadrilaterals; (C) use properties to classify three-dimensional figures, including pyramids, cones, prisms, and cylinders; and *(D) use critical attributes to define similarity.*		
7.7 Geometry and spatial reasoning. The student uses coordinate geometry to describe location on a plane.		

(A) locate and name points on a coordinate plane using ordered pairs of integers; and *(B) graph reflections across the horizontal or vertical axis and graph translations on a coordinate plane.* 7.8 Geometry and spatial reasoning. The student uses geometry to model and describe the physical world. *(A) sketch three-dimensional figures when given the top, side, and front views;* *(B) make a net (two-dimensional model) of the surface area of a three-dimensional figure; and* *(C) use geometric concepts and properties to solve problems in fields such as art and architecture.* 7.9 Measurement. The student solves application problems involving estimation and measurement. **(A) estimate measurements and solve application problems involving length (including perimeter and circumference) and area of**	A dining room is 12 feet by 15 feet. Design a circular table, so that twelve chairs, each 18 inches wide, will fit around the table. Allow at least 8 inches between the chairs. Explain how you determined the dimensions of the table. Draw a scale model of the dining room and your table. Is the table a reasonable size? Is there sufficient room for people to sit comfortably at this table in this room?	

polygons and other shapes; (B) connect models for volume of prisms (triangular and rectangular) and cylinders to formulas of prisms (triangular and rectangular) and cylinders; and (C) estimate measurements and solve application problems involving volume of prisms (rectangular and triangular) and cylinders.		
7.10 Probability and statistics. The student recognizes that a physical or mathematical model can be used to describe the experimental and theoretical probability of real-life events. *(A) construct sample spaces for simple or composite experiments; and* *(B) find the probability of independent events.* 7.11 Probability and statistics. The student understands that the way a set of data is displayed influences its interpretation. *(A) select and use an appropriate representation for presenting and*		

displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection; and *(B) make inferences and convincing arguments based on an analysis of given or collected data.* 7.12 Probability and statistics. The student uses measures of central tendency and range to describe a set of data. (A) describe a set of data using mean, median, mode, and range; and *(B) choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation.*		
7.13 Underlying processes and mathematical tools. The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines and activities inside and outside of school. (A) identify and		

apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics; (B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness; (C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and (D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems. 7.14 Underlying processes and mathematical tools. The student communicates about Grade 7 mathematics		

through informal and mathematical language, representations, and models. (A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and (B) evaluate the effectiveness of different representations to communicate ideas.		
7.15 Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. (A) make conjectures from patterns or sets of examples and non-examples; and (B) validate his/her conclusions using mathematical properties and relationships.		

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TEKS: Grade 8

Read an introduction to Texas Essential Knowledge and Skills charts.

TEKS	Examples	Commentary
111.24 MATHEMATICS, GRADE 8 8.1 Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations. (A) compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals; (B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships; (C) approximate (mentally and with calculators) the value of irrational numbers as they arise from problem situations (π, $\sqrt{2}$); and		
(D) express numbers in scientific notation, including negative exponents, in appropriate problem situations.		Negative exponents should be introduced as a way of expressing division.
8.2 Number, operation, and		

quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. (A) select appropriate operations to solve problems involving rational numbers and justify the selections; (B) use appropriate operations to solve problems involving rational numbers in problem situations; (C) evaluate a solution for reasonableness; and *(D) use multiplication by a constant factor (unit rate) to represent proportional relationships.* 8.3 Patterns, relationships, and algebraic thinking. The student identifies proportional or non-proportional linear relationships in problem situations and solves problems. (A) compare and contrast proportional and non-proportional linear relationships; and		
(B) estimate and find solutions to application problems involving percents and other proportional relationships such as similarity and rates.	1. Josh's father took him and four of his friends to San Antonio to watch a Spurs basketball game. The 126-mile drive took him 1 hour and 40 minutes and used 5 gallons of gas for a cost of $5.85. Use unit analysis to calculate each of the following for the entire trip. Make sure you show all your work. a. miles per hour b. miles per gallon c. dollars per hour d. dollars per gallon e. dollars per passenger	Students should write a sentence (using correct units) interpreting the meaning of each of their answers. For c, for example, a student might write, “This is the amount of money, in dollars, it takes to pay for one hour of their trip.”

<table>
<tr><td></td><td>f. yards per second
g. feet per second
h. cents per mile
i. cents per minute
j. miles per dollar
k. gallons per hour
l. passenger miles per gallon
m. cents per passenger miles</td><td></td></tr>
<tr><td>8.4 Patterns, relationships, and algebraic thinking. The student makes connections among various representations of a numerical relationship.</td><td>
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<tr><th>Figure Number</th><th>Number of Tiles</th></tr>
<tr><td>1</td><td>1</td></tr>
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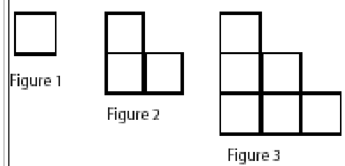

a. Using the same pattern, how many tiles would be needed to build figure 10? Explain.
b. Construct a graph using your table. Be sure to label and scale your axes.
c. Write an equation that will give you the number of tiles for any given figure number.</td><td></td></tr>
<tr><td>8.5 Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems.</td><td></td><td></td></tr>
<tr><td>(A) predict, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations; and</td><td>U.R.Online charges $17 per month plus 35 cents per hour for Internet service. Complete a table to show possible costs.</td><td></td></tr>
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<tr><th>Hours</th><th>Process</th><th>Total Cost</th><th>Cost per Hour</th></tr>
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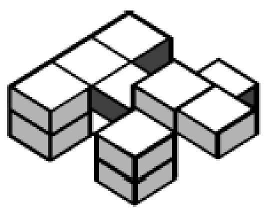
	| | | | | |---|---|---|---| a. What are the only values that are appropriate for the first column? b. Graph the data that compares the hours and cost per hour. Describe the graph as the number of hours increase. For which values of hours does it cost less than a dollar per hour to use the Internet? c. Write an equation that describes the monthly cost of Internet service in terms of hours used. d. If you are allowed to spend no more than $20 on Internet service, what is the maximum number of hours you are allowed on the Internet? Is this sufficient or do you need to look for another Internet provider? Explain your answer.	
(B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change) 8.6 Geometry and spatial reasoning. The student uses transformational geometry to develop spatial sense. *(A) generate similar figures using dilations including enlargements and reductions; and* *(B) graph dilations, reflections, and translations on a coordinate plane.*		
8.7 Geometry and spatial reasoning. The student uses geometry to model and describe the physical world. (A) draw three dimensional figures from different perspectives;		

(B) use geometric concepts and properties to solve problems in fields such as art and architecture; (C) use pictures or models to demonstrate the Pythagorean Theorem; and (D) locate and name points on a coordinate plane using ordered pairs of rational numbers. 8.8 Measurement. The student uses procedures to determine measures of three-dimensional figures. *(A) find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two dimensional models);* (B) connect models of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects; and *(C) estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.* 8.9 Measurement. The student uses indirect measurement to solve problems. *(A) use the Pythagorean Theorem to solve real-life problems;*		

and *(B) use proportional relationships in similar two-dimensional figures or similar three-dimensional figures to find missing measurements.*		
8.10 Measurement. The student describes how changes in dimensions affect linear, area, and volume measures.		
(A) describe the resulting effects on perimeter and area when dimensions of a shape are changed proportionally; and **(B) describe the resulting effect on volume when dimensions of a solid are changed proportionally.**	Students will build the unit dog using thirteen snap cubes and calculate his surface area and volume. Then students will work in pairs to construct from graph paper run on tag board, a dog with dimensions twice, triple, quadruple, or quintuple the original dog. They will calculate surface area and volume for their dog. The information for all groups will be collected, and the class will discuss the resulting effects on area and volume. 	The ratio of an animal's volume to surface area determines the biome in which they live. High ratios live in colder climates while lower ratios reside in warmer climates. This makes a nice interdisciplinary activity with a life science class.
8.11 Probability and statistics. The student applies concepts of theoretical and experimental probability to make predictions. *(A) find the probabilities of dependent and independent events;* *(B) use theoretical probabilities and experimental results to make predictions and decisions; and*		

(C) select and use different models to simulate an event.	Our assignment is to make a reasonable prediction about the number of goldfish crackers that are in a large bowl. This activity will use sampling techniques to estimate the population of a certain species within a defined area. Count out 200 goldfish crackers from your population of goldfish in your bowl and replace them with tagged (parmesan or pretzel) fish. Release these fish into the lake (bowl). Each team of students will use a net (small paper cup) to capture some fish. They will count the total number of fish in their sample and how many of the fish are tagged. They should return their fish to the lake before the next team takes their sample. Each team will estimate the total number of fish in the lake (bowl) using only their data, and then they will pool the class data to make another estimate. Students should give a written explanation about the differences between their estimates using only their data and then the class's pooled data.	The number of tagged fish needs to be between 10% and 30% of the total population in order to make an accurate prediction. In the video, *The Challenge of the Unknown*, there is a segment that deals with this experiment. The NCTM Addenda Series, Grades 5-8: *Understanding Rational Numbers and Proportions*, pages 57-60 describes this activity in detail.
(B) draw conclusions and make predictions by analyzing trends in scatter plots; and	Have students collect data on height vs. shoe size. Record the data in a table, plot the data on graph paper, and then determine a line of best fit. Since the same scale does not measure shoe sizes for males and females, you should separate the data by gender. After the data is collected and graphed, students should describe the data. What does the graph suggest? Is there a strong correlation between height and shoe size? If so, is the correlation positive or negative? Draw a line of best fit. Graph your height and shoe size on that line. How does your actual height and shoe size compare to the predicted shoe size for your height? Analyze the data using back-to-back stem plots or parallel box plots. What different information do these graphs provide?	A complete description of this activity is provided in the NCTM Addenda Series, Grades 9-12: *Connecting Mathematics*, pages 21-23, 26-29. This introduces the AP* Statistics concepts of correlation and linearity in bivariative data.
(C) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, circle graphs, bar graphs, box and whisker plots,		

histograms, and Venn diagrams with and without the use of technology.		
8.13 Probability and statistics. The student evaluates predictions and conclusions based on statistical data. (A) evaluate methods of sampling to determine validity of an inference made from a set of data; and *(B) recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.* 8.14 Underlying processes and mathematical tools. The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. (A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics; (B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for		

reasonableness; (C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing, and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and (D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems. 8.15 Underlying processes and mathematical tools. The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. (A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and (B) evaluate the effectiveness of different representations to communicate ideas. 8.16 Underlying processes and		

mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. (A) make conjectures from patterns or sets of examples and non-examples; and (B) validate his/her conclusions using mathematical properties and relationships.		

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TEKS: Algebra I

Read an introduction to Texas Essential Knowledge and Skills charts.

TEKS	Examples	Commentary
111.32 ALGEBRA I (ONE CREDIT) Foundations for functions: knowledge and skills and performance descriptions. (A.1) The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. (A) The student describes independent and dependent quantities in functional relationships. (B) The student gathers and records data, or uses data sets, to determine functional relationships between quantities. (C) The student describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.		

(D) The student represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities. (E) The student interprets and makes decisions, predictions, and critical judgements from functional relationships. (A.2) The student uses the properties and attributes of functions. (A) The student identifies and sketches the general forms of linear (y = x) and quadratic ($y = x^2$) parent functions. (B) The student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations, both continuous and discrete. (C) The student interprets situations in terms of given graphs or creates situations that fit given graphs *(D) The student collects and*		

organizes data, makes and interprets scatter plots (including recognizing positive, negative, or no correlation for data approximating linear situations), and models, predicts, and makes decisions and critical judgments in problem situations.		
(A.3) The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. (A) The student uses symbols to represent unknowns and variables. *(B) The student looks for patterns and represents generalizations algebraically.* (A.4) The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.		

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TEKS: Algebra II

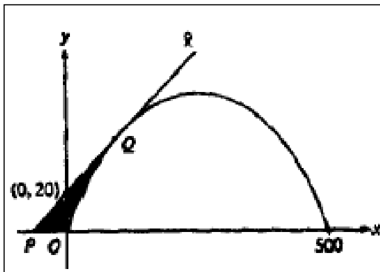
Read an introduction to Texas Essential Knowledge and Skills charts.

TEKS	Examples	Commentary
111.33 ALGEBRA II (ONE-HALF TO ONE CREDIT) Foundations for functions: knowledge and skills and performance descriptions. (2A.1) The student uses properties and attributes of functions and applies functions to problem situations. (A) identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations. *(B) collect and organize data, make and interpret scatter plots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgements.* (2A.2) The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. (A) use tools including factoring and properties of exponents to simplify expressions and to transform and solve		

equations. *(B) use complex numbers to describe the solutions of quadratic equations.* (2A.3) The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. (A) analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems. *(B) use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.* *(C) interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.* Algebra and geometry: Knowledge and skills and performance descriptions. (2A.4) The student connects algebraic and geometric representations of functions. (A) identify and sketch graphs of parent functions, including linear (f(x) = x), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x (f(x) = |x|), square root of x ($f(x) = \sqrt{x}$), and		

reciprocal of x(f(x) = 1/x).		
(B) extend parent functions with parameters such as a in f(x) = a|x and describe the effects of the parameter changes on the graph of parent functions.	Let f be the function given by $f(x) = x^3 - 6x^2 + p$, where p is an arbitrary constant. For what values of the constant p does f have 3 distinct real roots? Explain your reasoning.	This question is based on the 1997 AB4 AP* Calculus question.
(C) describe and analyze the relationship between a function and its inverse. (2A.5) The student knows the relationship between the geometric and algebraic descriptions of conic sections. (A) describe a conic section as the intersection of a plane and a cone. *(B) sketch graphs of conic sections, to relate simple parameter changes in the equation to corresponding changes in the graph.* *(C) identify symmetries from graphs of conic sections.* *(D) identify the conic section from a given equation.* *(E) use the method of completing the square.*		
Quadratic and square root functions: (2A.6) The student understands that quadratic functions can be represented in different ways and translates among their various representations.		

(A) determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities. *(B) relate representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions.* (C) determine a quadratic function from its roots or a graph. (2A.7) The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations. *(A) use characteristics of the quadratic parent function to sketch the related graphs and connect between the* $y = ax^2 + bx + c$ *and the* $y = a(x - h)^2 + k$ *symbolic representations of quadratic functions.* *(B) use the parent function to investigate, describe, and predict the effects of changes in a, h, and k on the graphs of* $y = a(x - h)^2 + k$ *form of a function in applied and purely mathematical situations.* (2A.8) The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.		

(A) analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems.	 A) Write the equation of the line shown in the figure. B) Write the equation for the parabola shown in the figure. C) Suppose the graph of the parabola shown in the figure represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along line shine on any part of the tree? Explain your reasoning.	This question is based on the 1996 AB6 AP Calculus question.
(B) analyze and interpret the solutions of quadratic equations using discriminants and solve quadratic equations using the quadratic formula. *(C) compare and translate between algebraic and graphical solutions of quadratic equations.*		
(D) solve quadratic equations and inequalities using graphs, tables, and algebraic methods.	Consider the curve defined by $x^2 + xy + y^2 = 27$. Solve for y in terms of x and use your calculator to graph the curve. Determine the intercepts algebraically and verify graphically.	This question is based on the 1994 AB3 AP Calculus question. Express the equation as a quadratic in terms of y, like this ($y^2 + xy + (x^2 - 27) = 0$). Use the quadratic b=x and $c=x^2 - 27$.
	Graph the region given by the quadratic system $25x^2 - 9y^2 \geq 225$, $x\geq0$ and $x\leq8$ on a coordinate plane. Rotate the region about the x-axis and sketch the resulting three-dimensional solid.	This introduces the calculus concept of solids of revolution. Also addresses TEKS 2A.8B and

		2A.8C.
(2A.9) The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. *(A) use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges.* *(B) relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions.* (C) determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities. *(D) determine solutions of square root equations using graphs, tables, and algebraic methods.* (E) determine solutions of square root inequalities using graphs and tables.		
(F) analyze situations modeled by square root functions, formulate equations or inequalities, select a method, and solve problems.	To start this activity you will need a stopwatch, a piece of string at least 1.7 meters long, a ruler, and a weight. 1. Tie the weight to the end of your string and then measure off 1.5 meters. 2. Have one person hold the pendulum. Use your stopwatch to time, in seconds, how long	This activity introduces the student to data collection, curve fitting, and experimental design—all topics of the AP Statistics

	it takes the pendulum to complete ten periods. Divide this time by 10 to estimate the time it takes for the pendulum to complete one period. 3. Repeat the process in part 2 several times, each time shortening the length of your string by 15 cm. Continue to collect your data (length of string, time/period). 4. Draw a sketch of the data, the time of a period as a function of the length of the pendulum. 5. If the graph appears linear, write an equation of the regression line that best models the data. If the graph does not appear to be linear, you will need to perform a transformation to straighten the data. What model and what transformation seem most appropriate? Write the equation of this model. 6. Predict t(.9 meters), that is, the time of one period if the length of the pendulum is .9 meters. 7. Predict t(2 meter), that is, the time of one period if the length of the pendulum is 2 meters. 8. Which of the two values t(.9) or t(2) do you feel is more accurate? Explain why. 9. What type of function did you determine was the best model for your data? Explain the process you had to use to re-express your data to be able to write an equation for this model.	curriculum. Does the weight of the pendulum affect the regression equation? Give each group a different weight and compare the results.
(G) connect inverses of square root functions with quadratic functions. (2A.10) The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.		
(A) use quotients of polynomials to describe the graphs of rational functions,	Let f be the function given by $f(x)= x / \sqrt{(x^2 - 4)}$ a. Find the domain of f.	This question is based on the 1989 AB4 AP Calculus question.

predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior.	b. Write an equation for each vertical asymptote to the graph of f. c. Write an equation for each horizontal asymptote to the graph of f.	
(B) analyze various representations of rational functions with respect to problem situations. (C) determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities. *(D) determine the solutions of rational equations using graphs, tables, and algebraic methods.* (E) determine solutions of rational inequalities using graphs and tables.		
(F) analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem.	The city of Katy, Texas, wants to enclose a 3000 square foot rectangular region as a park. The city plans to build a brick fence along 3 sides of the park that will cost $25 per linear foot. A wooden fence that will cost $10 per linear foot will enclose the fourth side of the park. Find the minimum cost of the fence.	This relates to the calculus concept of optimization.
(G) use functions to model and make predictions in problem situations, involving direct and inverse variation. (2A.11) The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the		

situation. **(A) develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses.**		
(B) use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior.	Let f be the function given by $f(x) = 2xe^{2x}$ Use your graphing calculator to do the following: a) find all horizontal asymptotes of f(x) b) locate the absolute minimum value of f c) determine the domain and range of f Examine the family of functions defined by $y = bxe^{bx}$ where b is a nonzero constant. What do all of these graphs have in common? Why?	This problem is based on the 1998 AB/BC2 AP Calculus exam. This is a good problem to tackle as a class. Every student can be assigned a different value of b.
(C) determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities. *(D) determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods.* (E) determine solutions of exponential and logarithmic inequalities using graphs and tables.		
(F) analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.	The rate of consumption of cola in the United States is given by $S(t) = Ce^{kt}$, where S is measured in billions of gallons per year and t is measured in years from the beginning of 1980.	This question is based on the 1996 AB/BC3 AP Calculus exam.

	a) The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find C and k. b) When did the consumption rate surpass 50 billion gallons per year?	

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TEKS: Geometry

Read an introduction to Texas Essential Knowledge and Skills charts.

TEKS	Examples	Commentary
111.34. GEOMETRY (ONE CREDIT) (G.1) Geometric structure: knowledge and skills and performance descriptions. The student understands the structure of, and relationships within, an axiomatic system. (A) The student develops an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems. (B) Through the historical development of geometric systems, the student recognizes that mathematics is developed for a variety of purposes. (C) The student compares and contrasts the structures and implications of Euclidean and non-Euclidean geometries. (G.2) The student		

analyzes geometric relationships in order to make and verify conjectures. (A) The student uses constructions to explore attributes of geometric figures and to make conjectures about geometric relationships. *(B) The student makes conjectures about angles, lines, polygons, circles, and three-dimensional figures and determines the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.* (G.3) The student applies logical reasoning to justify and prove mathematical statements. (A) The student determines the validity of a conditional statement, its converse, inverse, and contrapositive. (B) The student constructs and justifies statements about geometric figures and their properties. (C) The student demonstrates what it means to prove statements are true and find counter		

examples to disprove statements that are false. (D) The student uses inductive reasoning to formulate a conjecture. *(E) The student uses deductive reasoning to prove a statement.* (G.4) Geometric structure: The student uses a variety of representations to describe geometric relationships and solve problems. The student selects an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems. (G.5) Geometric patterns: The student uses a variety of representations to describe geometric relationships and solve problems.		
(A) use numeric and geometric patterns to develop algebraic expressions representing geometric properties. **(B) The student uses numeric and geometric patterns to make generalizations about geometric**	Use a graphing calculator to explore polygons with 3-12 sides using the unit circle (x = cos t, y = sin t) in parametric mode by adjusting the t-step values. To draw an n-sided figure, set the t-step to 360/n. Sketch the figure using graphing calculators, then transfer the figure by hand to polar paper. Determine the number of vertices, number of triangles formed by connecting one vertex to the others, sum of the angle measures, measure of each interior angle, measure of each exterior angle, sum of the measures of the exterior angles, number of diagonals, perimeter and area for a polygon with radius of 1. Generalize the pattern to write a formula for each of the explorations above for an n-gon. Also, notice how the perimeter values approach the circumference of a	Allow 1-2 class periods for this problem. AP* Calculus concept: Limits

properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles.	circle and how the area values approach the area of a circle.	
(C) The student uses properties of transformations and their compositions to make connections between mathematics and the real world in applications such as tessellations.		
(D) The student identifies and applies patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90) and (30-60-90) and triangles whose sides are Pythagorean triples.	If a 650 cm ladder is placed against a building at a certain angle, it just reaches a point on the building that is 520 cm above the ground. a) If the ladder is moved to reach a point 80 cm higher up, how much closer will the foot of the ladder be to the building? b) If the distance the ladder was moved inward is twice the distance it moved upward, how far is it from the wall? In right triangle ABC with right angle C, determine the measure of angles A and B using 30-60-90 or 45-45-90 ratios if $a = 3/\sqrt{2}$ and $b = 3\sqrt{6}$; if $a = 2$ and $c = 4$; if $a = 3\sqrt{2}$ and $c = 6$; etc. A boat is tied to a pier by a 25-foot rope. The pier is 15 feet above the boat. If 8 feet of rope is pulled in, how many feet will the boat move forward?	AP Calculus concept: Rates of Change
(G.6) Dimensionality and the geometry of location: The student analyzes the relationship between three-dimensional objects and related two-dimensional representations and uses these representations to solve problems.		

(A) The student describes and draws the intersection of a given plane with various three-dimensional geometric figures. (B) The student uses nets to represent and construct three-dimensional objects. (C) The student uses orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional figures and solve problems. (G.7) The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. (A) The student uses one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures. *(B) The student uses slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other*		

<table>
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<td>polygons.

(C) The student develops and uses formulas involving length, slope, and midpoint.

(G.8) Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.</td>
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<td>(A) The student finds areas of regular polygons, circles, and composite figures.</td>
<td>The rate at which Bethany's mailbox receives e-mails can be modeled by a continuous function. Selected values are shown in the chart below.

<table>
<tr><td>Time</td><td>8:00 AM</td><td>10:00 AM</td><td>12:00 PM</td><td>2:00 PM</td><td>4:00 PM</td></tr>
<tr><td>E-mail per Hour</td><td>5</td><td>7</td><td>10</td><td>9</td><td>12</td></tr>
</table>

(a) Sketch a scatter plot of the data. (b) Draw in 4 left-hand rectangles. (c) Describe the units represented by the dimensions of the rectangle. (d) Describe the units represented by the sum of the area of the rectangles. (e) Estimate the total e-mails Bethany received while she was at school using the 4 left-hand rectangles. (f) Draw in 4 right-hand rectangles using a dotted line. (g) Estimate the total e-mails Bethany received while she was at school using the 4 right-hand rectangles. (h) List the range of possible total e-mails using the answers to parts (e) and (g). (i) Geometrically demonstrate the error range of the 4 left- and 4 right-hand rectangles. (j) Create a new data table using the previous data, but showing every hour. (k) Compute the total e-mails Bethany received while she was at school using 8 left-hand rectangles. (l) Compute the total e-mails Bethany received while she was at school using 8 right-hand rectangles. (m) List the range of possible total e-mails using the answers to parts (k) and (l). (n) List the error range and compare it to your classmates'. (They should be the same regardless of the number chosen.) (o) Estimate the total e-mails received using the 4 trapezoids. (Note: a trapezoid is the average of the left- and right-hand rectangle.) (p) Estimate the total e-mails Bethany received while she was at school using the 2 midpoint rectangles.</td>
<td>AP Calculus Concept: Accumulation. See also AP Calculus 2000 AB2 and 99 AB3</td>
</tr>
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	Triangle ABC is inscribed in a semicircle centered at the origin with radius 3. Side AB of the triangle is on the x-axis and point C can be moved around the semicircle. (a) Sketch the problem situation. (b) Classify the triangle by angles. (c) Write the equation to graph the semicircle. (d) Determine the area of the triangle as a function of x. (e) List the domain for the problem situation. (f) Use a graphing calculator to determine the maximum area. Sketch the graph and justify your answers using increasing or decreasing functions and slope. (g) Determine the approximate dimensions that yield maximum area.	AP Calculus Concept: Optimization
	Draw a circle of radius 1 inscribed in a square. Simulate the throwing of a dart to determine the probability of hitting the circle by the use of random digits. To choose a point at random in the square, choose a pair of random digits (x,y) with the appropriate limits. If the pair of random digits lies within the circle, it is considered a hit. (a) Calculate the probability of hitting inside the circle. (b) Multiply the probability by 4. What number is represented? (c) Calculate the area of the circle divided by the area of the square and multiply by 4. What number is represented?	AP Statistics Concept: Randomization and probability
(B) The student finds areas of sectors and arc lengths of circles using proportional reasoning. **(C) The student derives, extends, and uses the Pythagorean Theorem.**	Point C is a point on a straight river. Town A is 11 miles straight across the river from C and Town B is 6 miles from that same river on the same side of the river as A. The distance from Town A to Town B is 13 miles. A pumping station is to be built along the river across from the towns at a point P to supply water to both towns. (a) Write an equation in terms of x, the distance from C to P, to express the total distance from A to P to B. (b) State the domain. (c) Use a graphing calculator to determine where the pumping station should be built in relationship to C so that the sum of the distances from the towns to the pumping station is a minimum. (d) Determine the minimum total distance. Sketch a graph and justify your answer using slopes of the curve, increasing and/or decreasing. (e) Determine the range of the distance function. (f) Using the minimum distance, determine how far the pumping station is from A and from B.	AP Calculus concept: Optimization
(D) The student finds surface areas and volumes of prisms, pyramids, spheres, cones, and cylinders, and composites of these figures in problem situations.	A sheet of metal is 60 cm wide and 10 m long. It is bent along its width to form a gutter with a cross section that is an isosceles trapezoid with 120-degree base angles. (a) Use 30-60-90 ratios to express the volume of the gutter as a function of the length of one of the equal sides. (b) For what value of x is the volume of the gutter a maximum—justify algebraically. (c) List the height and base lengths for the maximum volume. (d) Determine the maximum volume. (e) If the base angle is not known, use trig ratios to express the volume of the gutter as a function of the length of one of the equal sides and	AP Calculus concept: Optimization

	the base angle, q. (f) State the domain for q. (g) Using the x value obtained in part b, verify the volume in part d using the equation in part e with unit circle values.	
	A cylindrical soda can is designed to hold 7π cubic inches of soda (approximately 12 ounces). The material for the top and bottom costs $0.001 per square inch. The material for the vertical surfaces cost $0.0005 per square inch. (a) Sketch the problem situation. (b) Determine the height in terms of the radius. (c) Express the cost of materials used to make the can as a function of the radius. (d) Use a graphing calculator to determine the cost of the least expensive can. (e) At the minimum cost, how much would it cost the company to produce 1,000,000 cans? (f) Determine the dimensions of the least expensive can.	AP Calculus concept: Optimization
	Determine the volume of the solid formed by rotating the area of the region formed by: y = (-1/2)x + 1, y = 0, and x = 0 around the (a) x-axis, (b) y-axis, (c) x = -4	AP Calculus Concept: Volumes of Revolution. See also AP Calculus 2001 AB1, 99AB2, 98AB3, 97AB3, 96AB2
(G.9) The student analyzes properties and describes relationships in geometric figures. (A) The student formulates and tests conjectures about the properties of parallel and perpendicular lines based on exploration and using concrete models. (B) The student formulates and tests conjectures about the properties and attributes of polygons and their component parts based on exploration and using concrete models.		

(C) The student formulates and tests conjectures about the properties and attributes of circles and the lines that intersect them based on exploration and using concrete models. (D) The student analyzes the characteristics of polyhedra and other three-dimensional figures and their component parts based on exploration and using concrete models (G.10) The student applies the concept of congruence to justify properties of figures and solve problems. (A) The student uses congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane. *(B) The student justifies and applies triangle congruence relationships.* (G.11) Similarity and the geometry of shape. The student applies the concepts of similarity to justify properties of figures and solve problems.		

(A) The student uses similarity properties and transformations to explore and justify conjectures about geometric figures.		
(B) The student uses ratios to solve problems involving similar figures.	At a grain processing plant, the grain is falling off a conveyor and into a storage bin. The storage bin is the frustrum of a right cone with the larger base at the top. The smaller base at the bottom is closed while the grain is being poured into the bin, so the grain can be measured. When the bin is full, the contents are emptied. The top base has a radius of 3 feet, the bottom base has a radius of 2 feet. The bin has a height of 10 feet. If the height of the grain in the bin is 7.5 feet, what is the radius of the grain?	AP Calculus concept: Rates of Change
(C) The student develops, applies, and justifies triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods.	Determine the coordinates on the unit circle for multiples of 30 and 45 degrees for measures from 0 to 360 degrees using 30-60-90 and 45-45-90 ratios. Introduce the concept of radian measure in terms of arc length for 0 to 2π radians. A tightrope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A spotlight, at the top of the Jay building 70 feet above the tightrope, illuminates a tightrope walker. She is walking at a constant rate of 2 feet per second from the Jay building to the Tee building. (a) Sketch the problem situation. (b) Letting the distance she has walked on the tightrope be x and the letting the distance her shadow has moved along the ground be y, determine y in terms of x using a similar triangle. (c) How far from the Jay building is the tightrope walker when her shadow reaches the Tee building? (d) Where does her shadow move when it reaches the Tee building? (e) Determine the length of her shadow on the wall in terms of x using a new set of similar triangles.	This should be done as two separate lessons and emphasized continually throughout the course. Students should be able to produce the unit circle with measurements in degrees and radians by memory. AP Calculus Concepts: Related Rates of Change Adapted from AP Calculus 1991 AB6
(4) The student describes the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and applies this idea in solving problems.	Suppose cylinder A has twice the radius but half the height of cylinder B. How do the cylinder's lateral areas, total area and volumes compare? If the length and width of a rectangular solid are each decreased by 20%, by what percent must the height be increased for the volume to remain unchanged?	

[printer-friendly]

TEKS: Precalculus

Read an introduction to Texas Essential Knowledge and Skills charts.

TEKS	Examples	Commentary
111.35 PRECALCULUS (ONE HALF TO ONE CREDIT) (P.1) The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions. (A) describe parent functions symbolically and graphically, including • $f(x) = x^n$ • $f(x) = \ln x$ • $f(x) = \log_a x$ • $f(x) = 1/x$ • $f(x) = e^x$ • $f(x) = \|x\|$ • $f(x) = a^x$ • $f(x) = \sin x$ • $f(x) = \arcsin x$ etc.; (B) determine the domain and range of functions using graphs, tables, and symbols;		

(C) describe symmetry of graphs of even and odd functions;

(D) recognize and use connections among significant values of a function (zeros, maximum values, and minimum values, etc.), points on the graph of a function, and the symbolic representation of a function; and

(E) investigate the concepts of continuity, end behavior, asymptotes, and limits and connect these characteristics to functions represented graphically and numerically.

$$f(x)= \frac{6x^3+x^2+1}{2x-6}$$

a) Determine any and all points of discontinuity in f(x). Find algebraically and verify graphically. Use a window of $-10 \le x \le 10$ and $-50 \le y \le 300$.

b) Find the end behavior model and graph it and f(x) on the same axes using windows of $-20 \le x \le 20$, $-200 \le y \le 1000$ and $-30 \le x \le 30$ - $500 \le y \le 3000$.

c) Confirm the asymptotes and end behavior
i) graphically
ii) with tables

x	2	2.5	2.9	2.99	2.999	3	3.001	3.01	3.1	3.5	4
f(x)											

x	-1000	-100	-10	10	100	1000
f(x)						

	d) As x approaches infinity, what does f(x) approach? As x approaches negative infinity, what does f(x) approach? As x approaches 3 from the right, what does f(x) approach? As x approaches 3 from the left, what does f(x) approach?	AP* Calculus Concept: Limits
(P.2) The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems. (A) apply basic transformations, including • a · f(x) • f(x) + d • f(x - c) • f(b · x), and • compositions with absolute value functions, including |f(x)| and • f(|x|) to the parent functions; (B) perform operations including composition on functions, find inverses, and describe these procedures and results verbally, numerically, symbolically, and graphically, and		
(C) investigate identities graphically and verify them symbolically, including	f(x) = sin 2x g(x) = 2sin x cos x a) Put values for x, sin 2x, and 2sinxcosx, $-\pi \le x \le \pi$, in a table.	

<table>
<tr>
<td>logarithmic properties, trigonometric identities, and exponential properties.</td>
<td>b) Graph f(x) and g(x) on the same axes.

c) Verify that f(x) = g(x) at $x = \pi / 6$.

d) Use the properties of circular functions to prove that sin 2x = 2sin x cos x.</td>
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<tr>
<td>(P.3) The student uses functions and their properties, tools, and technology to model and solve meaningful problems.</td>
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<td>(A) investigate properties of trigonometric and polynomial functions;

(B) use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data;

(C) use regression to determine the appropriateness of a linear function to model real-life data (including technology to determine the correlation coefficient); and

(D) use properties of functions to analyze and solve problems and make predictions.</td>
<td>
<table>
<tr><th>Year</th><th>Fed Debt</th></tr>
<tr><td>1980</td><td>0.91</td></tr>
<tr><td>1981</td><td>0.99</td></tr>
<tr><td>1982</td><td>1.1</td></tr>
<tr><td>1983</td><td>1.4</td></tr>
<tr><td>1984</td><td>1.6</td></tr>
<tr><td>1986</td><td>2.1</td></tr>
<tr><td>1987</td><td>2.3</td></tr>
<tr><td>1988</td><td>2.6</td></tr>
<tr><td>1989</td><td>2.9</td></tr>
<tr><td>1990</td><td>3.2</td></tr>
<tr><td>1991</td><td>3.6</td></tr>
</table>
(Debt values are in trillions)

a) Draw a scatter plot of the data.

b) Calculate the equation of the line of best fit.

c) Use the equation from part (b) to predict the debt in 1985 as well as 1994.

d) The scatter plot appears to be a curve, suggesting exponential growth. Linearize the curved data by graphing a scatter plot of (year, ln debt).

e) Calculate the regression line for the re-expressed data.

f) Use properties of exponents/logarithms to rewrite the equation in part (e) into its exponential form.

g) Use the exponential equation in part (f) to predict the debt in 1985 and 1994.</td>
<td>This includes the AP* Statistics concepts of transformations to achieve linearity, specifically logarithmic and power transformations.</td>
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	h) Which of the two sets of predictions seem to be more accurate? Explain.	
(E) solve problems from physical situations using trigonometry, including the use of Law of Sines, Law of Cosines, and area formulas and incorporate radian measure where needed.		
(P.4) The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems.		
(A) represent patterns using arithmetic and geometric sequences and series;	Using (1,1), (2,5), (3,12), (4,22) ... a) What is the 10th term in the sequence above? b) Write an equation for the sequence of the ordinants. c) Graph a scatter plot of the first 10 terms. d) Graph the equation from part b on the same axes as the scatter plot. e) Find the sum of the heights (above the independent axis) of each of the first 10 points. f) Express the sum in part e using sigma notation. g) Approximate the area between the curve (connecting the 10 points) and the x-axis using trapezoids. h) If the ordered pairs represent (week, homework problems per week), what is the meaning of the area found in part g?	AP Calculus Concept: Accumulation 8
(B) use arithmetic, geometric, and other sequences	A patient has a throat infection and is to take 300 mg of an antibiotic every 6 hours. At the end of 6 hours, about 3% of the medication is still in the body.	AP Calculus Concept: Limits

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<td>and series to solve real-life problems;</td>
<td>
a) What quantity of medication is in the body immediately after the 4th dose?

b) What quantity of medication is in the body immediately after the 12th dose?

c) Assuming the patient continues taking the medication, what eventually happens to the drug level in the patient's body?

d) Show graphically and use a table of values to confirm the conclusion from part (c).

<table>
<tr><td>dose d</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>medication m</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
e) Use sigma notation to express the data as a series.

f) Complete the statement: "as the number of doses increases, ..."
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<td>(C) describe limits of sequences and apply their properties to investigate convergent and divergent series; and</td>
<td>
Given that

$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{n(n+1)}$

$= \frac{n}{n+1}$

a) Prove the given statement by induction.

b) Show that

$\lim_{n \to \infty} \frac{n}{n+c} = 1$

c) Find

$\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$ by finding the limit of the sequence of partial sums s_1, s_2, s_3, s_4, ...s_n.

d) Confirm the answer to part "c" by graphing the partial sums as n increases.

e) Verbalize what you learned; "As n
</td>
<td>These problems are particularly important for students who will be taking BC Calculus.</td>
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</table>

	increased, ..."	
(D) apply sequences and series to solve problems including sums and binomial expansion.	Suppose Tim Duncan of the San Antonio Spurs is a 77% free throw shooter. a) Use binomial expansion to determine the probability of Tim's making exactly 4 of 9 free throws in a game. b) Determine the probability of Tim's making at least 4 of 9 free throws in a game. c) Write (in the form of a chart) a probability distribution to determine the most likely outcome when Tim attempts 9 free throws. d) What is the probability of the most likely outcome? e) Draw a bar graph that represents the probability distribution of Tim's making 0 through 9 free throws.	AP Statistics concept: Discrete random variables and their probability distributions.
(P.5) The student uses conic sections, their properties, and parametric representations, as well as tools and technology, to model physical situations. (A) use conic sections to model motion, such as the graph of velocity vs. position of a pendulum and motions of planets; (B) use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound;		
(C) convert between parametric and rectangular forms of functions and equations to graph them; and	$x = 2\cos^2 t$ and $y = \sin 2t$ a) Graph the conic section formed by x and y with your calculator set in parametric mode. b) Using properties of circular functions, find a	AP Calculus Concept: Parametric equations are used to describe motion.

	rectangular equation for the curve that contains no variables other than x and y. c) Graph the equation from part b above. d) Select values of t so that the graph of the parametric equations begins at (1,1) and ends at (2,0). Show a table of values and a graph that confirms your work. e) Rewrite the parametric equations and choose values of t so that it graphs clockwise, starting and ending at (1,1). Show a table of values and a graph that confirms your work.	Students find parametric rates of change and determine velocity, acceleration, and arc length.
(D) use parametric functions to simulate problems involving motion.	A circle with equation $(x+2)^2 + y^2 = 9$ is intersected by a graph with parametric equations $x = 2\cos^2 t$ and $y = \sin 2t$. a) Convert the circular equation above to parametric form. Convert the parametric equations to rectangular form. b) Find the points of intersection between the circle and the other graph. c) Let t represent the time, in seconds, of an object moving along the conic sections above. Will an object on the 1st conic section reach the 1st intersection point before, after, or at the same time as an object moving on the second conic section? Determine algebraically and confirm graphically. d) Find the minimum distance between objects moving along the conic sections where $0 \le t \le 1$. At what time does the minimum distance occur?	AP Calculus Concept: Optimization
(P.6) The student uses vectors to model physical situations. *(A) use the concept of vectors to model situations defined by magnitude and direction; and* *(B) analyze and solve vector problems generated by real-life situations.*		

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